A DFT based low complexity LMMSE channel estimation technique for OFDM system

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Abstract — The linear minimum mean square error (LMMSE) channel estimation technique is often employed for orthogonal frequency division multiplexing (OFDM) system because of its optimal performance in the mean square error (MSE) perspective. However, the LMMSE method requires cubic complexity of order $O(N_p^3)$ where N_p is the number of pilot subcarriers. To reduce the computational complexity, a discrete Fourier transform (DFT) based LMMSE method is proposed in this paper for OFDM system in frequency selective channel. To validate the proposed method, the closed form mean square error (MSE) expression is also derived. Finally, the computer simulation is carried out to compare the performance of proposed LMMSE method with the classical LS and LMMSE methods in terms of bit-error-rate (BER) and computational complexity.The simulation results show that the proposed LMMSE method achieves exactly same performance as conventional LMMSE method with much lower computational complexity.

Keywords* — *Channel estimation, OFDM, LMMSE, Mean square error

1. Introduction

Orthogonal frequency division multiplexing (OFDM) has attracted a lot of attentions due to its high spectrum efficiency, fast and easy implementation using fast Fourier transformation (FFT). It is also resilient against inter symbol interference (ISI) which occurs due to the frequency selective fading channel [1]. However, high peak-to-average power ratio (PAPR) [2] and channel estimation accuracy [3] are the major challenges for OFDM systems. The equalization of the OFDM system solely depends on the accuracy of the channel estimation [4]. Generally, there are two schemes to estimate the channel namely: non-blind or pilot aided and blind schemes. As compared to pilot aided channel estimation, the blind channel estimation is limited to slow time varying channel and also has higher complexity and poorer performance. Hence, pilot aided channel estimation is preferred over blind channel estimation. Based on comb-type pilot, the least square (LS) and minimum mean square error (MMSE) based channel estimation methods have been investigated in [5]. The LS estimation has low computational complexity but its mean square error (MSE) is high due to noise enhancement problem. To obtain better performance of the LS based estimation method, several denoising strategies have been proposed in [6, 7, 8, 9]. An eigen-select denoising threshold [6], linear filtering least square method [7], AdaBoost [8] and singular spectrum analysis (SSA) [9] based channel estimation technique are proposed for channel estimation in OFDM system. In [10], the authors proposed an adaptive SSA based channel estimation method. In adptive SSA, additional noise reduction is performed at singular value level and therefore, it provides better performance as compared to SSA algorithm bassed channel estimation method. However, all these channel estimation methods provide trade-off between performance and computational complexity. If the power delay profile (PDP) of the channel is aprior known to the receiver, the LMMSE channel estimation method is typically implemented. However, it requires cubic complexity due to matrix inversion operation. To reduce the complexity of LMMSE estimation, a low rank approximation based singular value decomposition (SVD) is proposed in [11]. Based on SVD method [11], the authors proposed two efficient channel estimation methods for OFDM/OQAM system in [12]. However, these SVD based estimation methods require still high computational complexity as decomposing the R_{HH} matrix using SVD method itself requires cubic complexity [13]. In [14], the authors approximate the LMMSE method using the law of large numbers to reduce the computational complexity of the channel to $O(N \log N)$. However, its performance is poor at high SNR value. In [15], the authors proposed a Dual-Diagonal LMMSE (DD-LMMSE) channel estimation method with $O(N \log N)$ and also derive the closed form expression of the asymptotic MSE of the DD-LMMSE. In [16], the author proposed a low complexity LMMSE channel estimation method based on K terms Neumann series expansion method to avoid the matrix inversion. A joint low complexity channel estimation and symbol detection is proposed in [17] based on message passing algorithm. In this method, the Sherman-Morrison formula is applied which transforms the cubic matrix inversion into a series of diagonal matrix inversions thus reduces the computational complexity. In [18],the authors proposed a Conjugant Gradient (CG) based channel estimation to achieve similar performance as LMMSE method. This method performs the channel estimation in an iterative manner and requires computational complexity of the order $O((N_p \log N_p)G)$ where G is the number of iterations. Typically a high value of G is required to obtain the optimal performance. A structure based LMMSE estimation method was proposed in [19] by assuming that the number of pilots is an exact integer multiple of channel length. This method depends on a-prior information of the channel impulse response and the appropriate placement of pilots across the OFDM subcarriers. Although the length of CIR can be obtained from the knowledge of the channel autocorrelation function by using adaptive guard interval (GI) as given in [20], but, the length of the channel may not guaranteed to be an exact integer multiples of number of pilot subcarriers. A compressed sensing (CS) algorithm based MMSE channel estimation is proposed in [21]. This method provides similar performance to LMMSE with much lower computational complexity. However, this method assumes that the channel coefficients are sparse while estimating the channel. Recently, an LMMSE algorithm based on vector quantization approach was proposed in [22] for OFDM system. In this method, first the LMMSE filtering matrices corresponding to wireless channel parameters are calculated offline. Subsequently, an appropriate LMMSE filtering matrix is selected according to the MSE criterion while estimating the channel in online mode. Therefore, this method does not require the PDP to be known at the receiver and hence provides negligible performance degradation with much lower computational complexity.

In this paper, a very simple but efficient LMMSE channel estimation technique is proposed by exploiting discrete Fourier transformation (DFT) and circulant properties of the channel frequency autocorrelation matrix (R_{HH}) for OFDM system in frequency selective fading channel.

The symbols associated with matrices and vectors are denoted in boldface and underline alphabets, respectively. The notations $(.)^H$, $(.)^{-1}$ denote the Hermitian and inverse operation, respectively. Similarly, $(.)_p$ denotes the position of pilot signal and $E[.]$ symbolises the expectation operator.

2. System Model

Let us consider an OFDM system with N number of subcarriers. After some signal manipulation such as addition of cyclic prefix (CP), removal of CP, inverse fast Fourier transformation (IFFT) and FFT operation, the received signal vector in the frequency domain is given by

$$\underline{Y} = \mathbf{X}\,\underline{H} + \underline{W} \tag{1}$$

where $\underline{Y} = [Y(0), Y(1), ..., Y(N-1)]^T$. The transmitted signal $\mathbf{X} = diag[X(0), X(1), ..., X(N-1)]$ is an $N \times N$ diagonal matrix. The symbol $\underline{H} = [H(0), H(1), ..., H(N-1)]^T$ and $\underline{W} = [W(0), W(1), ..., W(N-1)]^T$ are the $N \times 1$ channel frequency response (CFR) and additive white Gaussian noise (AWGN) vector, respectively. In this paper, the comb-type pilot pattern is adopted for channel estimation purpose. After extraction of pilot symbol at the receiver side, the received signal vector at the pilot position can be written as

$$\underline{Y}_p = \mathbf{X}_p \underline{H}_p + \underline{W}_p \tag{2}$$

The parameter $\underline{Y}_p$, $\mathbf{X}_p$ and $\underline{H}_p$ are the frequency domain received signal, transmitted signal and CFR at the pilot position, respectively. The CFR vector at pilot subcarrier can be represented as

$$\underline{H}_p = \boldsymbol{F}_p \underline{h} \tag{3}$$

where $\boldsymbol{F}_p$ is an $N \times L$ unitary FFT matrix with $\boldsymbol{F}_p(k,l) = e^{-j2\pi kl/N}$, $k = 0 : p_s : (N_p - 1)p_s$, $l = 0 : L-1$. The IFFT matrix is defined as $F_p^H = \frac{1}{N_p}(F_p)^H$. The channel impulse response (CIR) vector is defined as $\underline{h} = [h(0), h(1), ..., h(L-1)]$ where L is the total number of multipath channels. It is assumed that, each multipath channel $h(l)$ are independent and identically distributed (i.i.d.) with zero mean complex Gaussian random variable. The corresponding autocorrelation of the CIR $\underline{h}$ is given by $E[\underline{hh}^H] = \Delta$ with $\Delta = diag(\Lambda(0), \Lambda(1), ..., \Lambda(L-1))$ is the diagonal PDP matrix and $\Lambda(l)$ denotes the average power of the lth delay path. The total power of the PDP $tr(\Delta) = 1$ where $tr(.)$ denotes the trace operation. The power delay profile (PDP) of the multipath is aprior known to the receiver or it can be calculated using the method given in [23] with a computational complexity of $O(L^2)$. The channel estimation using LS criterion is given by

$$\tilde{\underline{H}}_{p,ls} = (X_p)^{-1} \underline{Y}_p \tag{4}$$

In order to obtain the channel at all data subcarrier, the interpolation methods are to be performed such as linear interpolation, low pass interpolation, discrete Fourier transform (DFT) based interpolation and so on [24]. In this paper, the DFT based interpolation is adopted to obtain the CFR at all data subcarriers. After estimating the channel at all data subcarriers, one tap zero forcing equalization is performed to obtain the desired transmitted data signal at the receiver side. The LS method suffers from high MSE and thus the LMMSE channel estimation method is typically adopted which is optimal in the MSE perspective.

From [5], the LMMSE channel estimation method at pilot positions can be written as

$$\tilde{\underline{H}}_{p,lmmse} = \boldsymbol{R}_{\underline{H}_p \underline{H}_p} (\boldsymbol{R}_{\underline{H}_p \underline{H}_p} + (\beta/SNR)\boldsymbol{I}_{Np}))^{-1} \tilde{\underline{H}}_{p,ls} \tag{5}$$

The signal to noise ratio (SNR) is defined as $SNR = E[|\underline{X}_p|^2]/\sigma_W{}^2$. The symbol $\beta = E[|\underline{X}_p|^2]/E[|1/|\underline{X}_p|^2]$ is a constant depending on the constellation. For QPSK and 16QAM modulation, the value of β are 1 and 17/9, respectively. The LMMSE method experiences high computational complexity of the order $O(N_p^3)$ due to the matrix inversion operations as given in (5).

3. Proposed low complexity LMMSE method

In this section, a low complexity LMMSE method is proposed by exploiting the DFT technique. The channel frequency autocorrelation matrix is the Fourier transformation of the PDP and is given as

$$\boldsymbol{R}_{\underline{H}_p \underline{H}_p} = E[\underline{H}_p \underline{H}_p^H] = N_p \boldsymbol{F}_p \Delta \boldsymbol{F}_p^H. \tag{6}$$

As $\boldsymbol{R}_{\underline{H}_p\underline{H}_p}$ and $\boldsymbol{R}_{\underline{H}_p\underline{H}_p} + (\beta/SNR)\boldsymbol{I}_{Np}$ are circulant matrices, hence they are commutative [25]. Thus, the LMMSE method can be rewritten as

$$\tilde{\underline{H}}_{p,lmmse} = (\boldsymbol{R}_{\underline{H}_p\underline{H}_p} + (\beta/SNR)\boldsymbol{I}_{Np})^{-1}\boldsymbol{R}_{\underline{H}_p\underline{H}_p}\tilde{\underline{H}}_{p,ls} \quad (7)$$

Multiplying both sides of (7) with $(\boldsymbol{R}_{\underline{H}_p\underline{H}_p} + (\beta/SNR)\boldsymbol{I}_{Np})$matrix, we have

$$(\boldsymbol{R}_{\underline{H}_p\underline{H}_p} + (\beta/SNR)\boldsymbol{I}_{Np})\tilde{\underline{H}}_{p,lmmse} = \boldsymbol{R}_{\underline{H}_p\underline{H}_p}\tilde{\underline{H}}_{p,ls} \quad (8)$$

As (β/SNR) is a scalar quantity, then $(\beta/SNR)\boldsymbol{I}_{Np}$ can be written as $N_pF_p(\frac{1}{N_p}(\beta/SNR)I_{\mathbf{L}})F_p^H$ and (8) can be simplified to

$$\begin{aligned}
&[N_pF_p\Delta F_p^H + N_pF_p(\tfrac{1}{N_p}(\tfrac{\beta}{SNR})\boldsymbol{I}_L)F_p^H]\tilde{\underline{H}}_{p,lmmse} \\
&\qquad = N_pF_p\Delta F_p^H\tilde{\underline{H}}_{p,ls} \\
&\Rightarrow N_pF_p(\Delta + \tfrac{1}{N_p}(\tfrac{\beta}{SNR})\boldsymbol{I}_L)F_p^H\tilde{\underline{H}}_{p,lmmse} = N_pF_p\Delta F_p^H\tilde{\underline{H}}_{p,ls} \\
&\Rightarrow (\Delta + \tfrac{1}{N_p}(\tfrac{\beta}{SNR})\boldsymbol{I}_L)\tilde{\underline{h}}_{lmmse} = \Delta\tilde{\underline{h}}_{ls} \\
&\Rightarrow \tilde{\underline{h}}_{lmmse} = \delta\tilde{\underline{h}}_{ls}
\end{aligned} \quad (9)$$

where $\delta = diag[\frac{\Lambda(0)}{\Lambda(0)+\frac{1}{N_p}(\frac{\beta}{SNR})}, \ldots, \frac{\Lambda(L-1)}{\Lambda(L-1)+\frac{1}{N_p}(\frac{\beta}{SNR})}]$. The parameters $\tilde{\underline{h}}_{ls}$ and $\tilde{\underline{h}}_{lmmse}$ are the estimated channel in the time domain using LS and MMSE criterion, respectively. The estimated channel frequency response is given as $\tilde{\underline{H}}_{lmmse} = \boldsymbol{F}_l\tilde{\underline{h}}_{lmmse}$, where $\boldsymbol{F}_l$ is an $N \times L$ FFT unitary matrix.

4. Performance Analysis

In this section, the mean square error of the proposed DFT based LMMSE channel estimation method is derived. The time domain LS method is given by $\tilde{\underline{h}}_{ls} = F_p^H\tilde{\underline{H}}_{p,ls} = \underline{h} + F_p^H(X_p)^{-1}W_p$. The mean square error of the proposed channel estimation method can be written as

$$\begin{aligned}
mse &= \tfrac{1}{L}trE[|\underline{h} - \tilde{\underline{h}}_{lmmse}|^2] \\
&= \tfrac{1}{L}trE[|\underline{h} - \delta(\underline{h} + F_p^H(X_p)^{-1}W_p)|^2] \\
&= \tfrac{1}{L}trE[\underline{hh}^H - \underline{hh}^H\delta^H - \delta\underline{hh}^H \\
&\quad + \ \delta\underline{hh}^H\delta^H + \delta F_p^H\tfrac{\beta}{SNR}I_{N_p}F_p\delta^H] \\
&= \tfrac{1}{L}(\sum_{l=0}^{L-1}\delta(l)[1-\delta(l)]^2 + \delta(l)^2\tfrac{1}{N_p}\tfrac{\beta}{SNR})
\end{aligned} \quad (10)$$

where $\delta(l) = \frac{\Lambda(l)}{\Lambda(l)+\frac{1}{N_p}(\frac{\beta}{SNR})}$ and the parameter $\Lambda(l)$ is power of the lth multipath channel.

5. Computational Complexity

The efficacy of LS, LMMSE and proposed method can be compared by evaluating the computational complexity in obtaining the CFR at all subcarriers. The LS estimation technique at pilot positions given in (4) requires N_p number of complex multiplications. In order to obtain the CFR at for all subcarriers, the DFT interpolation is used. This requires N_p numbers of IFFT and N number of FFT operations. Therefore, the overall computational complexity of LS estimation requires $N_p + N_p\log_2 N_p + N\log_2 N$. The conventional LMMSE channel estimation at pilot positions as given in (5) requires multiplication and inversion of CFR autocorrelation matrix ($\boldsymbol{R}_{\underline{H}_p\underline{H}_p}$) of size $N_p \times N_p$. In order to determine the $\boldsymbol{R}_{\underline{H}_p\underline{H}_p}$ matrix in (6) requires N_p point FFT / IFFT operations of the diagonal channel autocorrelation matrix $E[hh^H]$. As a result, the computation complexity of $\boldsymbol{R}_{\underline{H}_p\underline{H}_p}$ is $N_p\log_2 N_p$. Thus, the LMMSE channel estimation at pilot positions requires $N_p^2 + N_p^3 + N_p\log N_p$ computational complexity. Similarly, the CFR at all subcarriers as discussed in the previous paragraph requires Np log Np + N log N operations. The overall computational complexity to obtain CFR using LMMSE technique requires $N_p^2 + N_p^3 + 2N_p\log_2 N_p + N\log_2 N$. The proposed DFT based low complexity LMMSE method is given in (9). The δ in (9) requires diagonal matrix inversion of size $L \times L$. Therefore, the calculation of δ parameter needs L number of complex multiplications. The CFR at all subaccriers can be obtained by N point FFT operations. Therefore, the overall computational complexity of proposed low complexity LMMSE method requires $L + N\log_2 N$ operations. The overall computational complexity of various channel estimation methods are compared and is listed in Table 1.

Table 1
Computational Complexity

Methods	Computational Complexity
LS	$N_p + N_p\log_2 N_p + N\log_2 N$
Classical LMMSE	$N_p^2 + N_p^3 + 2N_p\log_2 N_p + N\log_2 N$
Proposed LMMSE	$L + N\log_2 N$

Table 2
Simulation Parameters

Parameters	Value
Number of Subcarriers	128
Number of FFT	128
Number of CP	16
Modulation Type	QPSK/16QAM
System Bandwidth	1MHz
Subcarrier Spacing	7.8125 KHz
Channel Type	Exponential Decaying PDP
Pilot Spacing	8
Normalized delay spread	1.5
Number of multipath	14

6. Simulation Results

In this section, the performance of proposed LMMSE method is compared with classical LS and LMMSE channel estimation methods in terms of mean-square error (MSE) and bit error rate (BER). In this paper, the following parameters are considered for OFDM system model: the number of subcarriers N = 128, length of cyclic prefix N_{CP}=16, system bandwidth B = 1MHz and QPSK / 16QAM modulation. If no modulation scheme is specified in simulation, then it is considered to be 16QAM modulation scheme. The channel is assumed to be exponential decaying power delay profile (PDP) [26] with power of the l-th path is given as $\Lambda(l) = \Lambda(0)e^{-l/d}$ where $\Lambda(0) = 1 - e^{-1/d}/1 - e^{-L/d}$ is the power of the first multipath channel. The parameter $d = -\tau_{rms}/T_s$ is the normalized delay spread where T_s is the sampling period and τ_{rms} is the root mean squared (rms) delay of the channel. The number of multipath is given by $L = \tau_{\max}/T_s$ where τ_{max} is the maximum excess delay and is defined as $\tau_{\max} = \tau_{rms}\ln A$ with A being the ratio of non-negligible path power to first path power. In this paper, the value of A is taken as A = -40dB and normalized delay spread d =1.5 and this leads to total number of multipath L = 14. The total number of pilots are considered as $N_p = 16$ with pilot spacing p_s = 1:8. The total simulation parameters is listed in the Table 2. Prefect time and frequency synchronization are assumed at the receiver side.

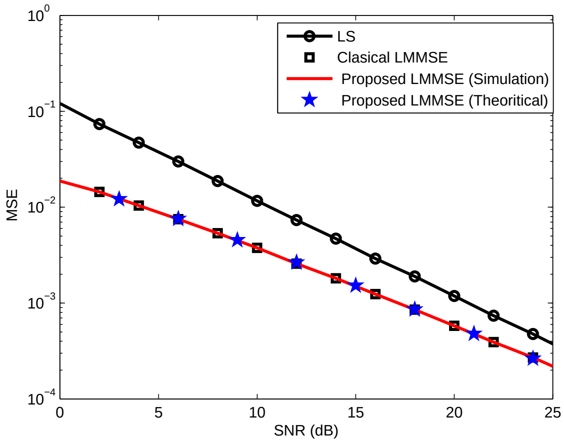

Fig. 1. MSE vs SNR performance comparison of proposed LMMSE, classical LS and LMMSE channel estimation methods

Fig. 1. shows the MSE performance with respect to SNR for the proposed LMMSE, classical LS and LMMSE methods with normalized delay spread $d = 1.5$ and L = 14. The simulation result shows that the LS method has poorer MSE performance as compared to LMMSE method due to noise enhancement problem. It is observed that the MSE performance of the proposed LMMSE estimation method are exactly matched with the classical LMMSE method. This is due to the fact that, the proposed method is directly derived from the classical LMMSE method without any approximations.

In order to analyze the effect of normalized delay spread d on the performance of various channel estimation methods, multiple values of d are taken into considerations e.g. $d = [0.3\ \ 0.6\ \ 0.9\ \ 1.2\ \ 1.5\ \ 1.8]$. This leads to total number of multipath channel as $L = [3\ \ 6\ \ 8\ \ 12\ \ 14\ \ 16]$. The MSE vs normalized delay spread (d) for various channel estimation methods is shown in the Fig. 2. The result is obtained for pilot spacing $p_s = 8$ at 25dB SNR. The simulation result shows that the performance of the proposed LMMSE is exactly matched with classical LMMSE irrespective of any value of normalized delay spread d. It is also noticed that

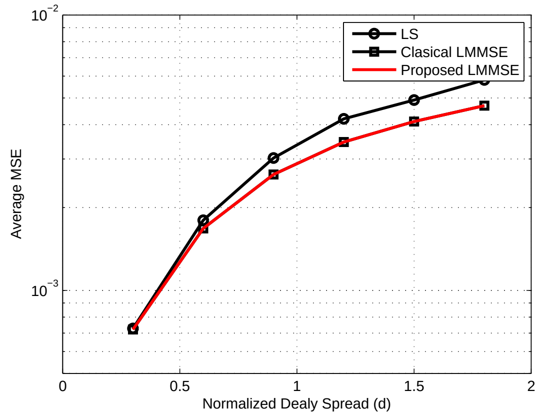

Fig. 2. MSE vs normalized delay spread of proposed LMMSE, classical LS and LMMSE channel estimation methods

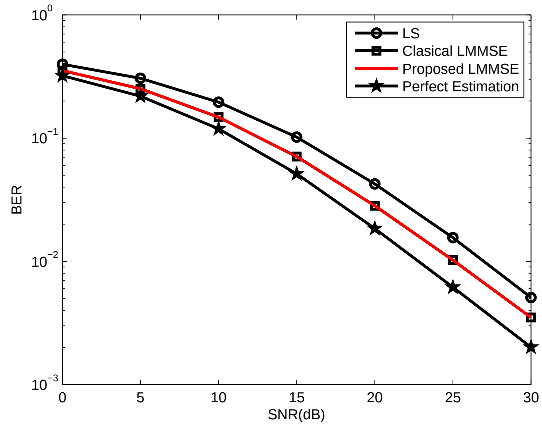

Fig. 3. BER vs SNR performance comparison of proposed LMMSE, classical LS and LMMSE channel estimation methods for pilot spacing $p_s = 8$

the performance of LS is close to LMMSE method for low value of d. However, the performance gap increases with increase in normalized delay spread value.

The BER performance comparison of proposed LMMSE, classical LS, LMMSE and perfect channel estimation method for pilot spacing $p_s = 8$ is shown in the Fig. 3. The simulation results show that the BER performance of the LS estimation method is poorer as compared to LMMSE estimation method. From the Fig 3, it is observed that the performance of the proposed LMMSE method is very close to perfect estimation method where it is assumed that the complete channel state information(CSI) is known at the receiver side.

The BER vs SNR performance comparison of various channel estimation methods with QPSK and 16QAM modulation for pilot spacing $p_s = 8$ is shown in the Fig 4. From, the simulation results, it is seen that, the performance of QPSK modulated channel estimation methods outperform the 16QAM modulated channel estimation methods. It is also observed that, the performance gap between the LMMSE and perfect estimation with QPSK modulation is closer as compared to 16QAM modulation.

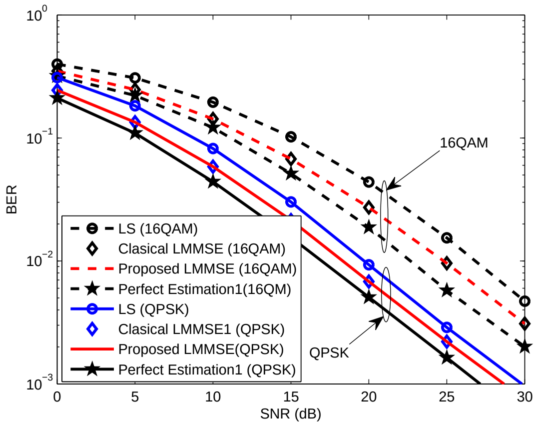

Fig. 4. BER vs SNR performance comparison of proposed LMMSE, classical LS and LMMSE channel estimation methods with QPSK and 16QAM Modulation for pilot spacing $p_s = 8$

7. Conclusion

In this paper, an optimal low complexity DFT based LMMSE channel estimation method is proposed for OFDM system in frequency selective channel. The closed form MSE expression is also derived for this proposed method to validate the proposed method. The proposed LMMSE method is compared with conventional channel estimation methods in terms of performance and computational complexity. The simulation results show that proposed LMMSE channel estimation has exactly matched with its theoretical value and achieves the same performance as the classical LMMSE channel estimation with only $L+N\log_2 N$ computational complexity. The limitation of the proposed LMMSE channel estimation technique is that it is not applicable to non-sample spaced channel as the proposed LMMSE method utilizes DFT technique to estimate the channel.

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