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# Does the inclusion of exposure to volatility into diversified portfolio improve the investment results? Portfolio construction from the perspective of a Polish investor[1]

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**Abstract**: The main goal of this research is to analyse the investment benefits from an incorporation of the volatility exposure to the diversified portfolio from the perspective of a Polish investor. Volatility, treated as a new asset class, may improve the performance of the portfolio due to its negative correlation with most types of assets. This topic has been widely investigated for the United States and Europe whereas the Polish market appears to be not heavily researched and this study may fill this gap. The research covers the period from October 2010 to July 2018 and is performed on daily close prices. To construct the portfolios the analysis uses the mean-variance framework and the naïve diversification approach. The comparison of risk-adjusted returns between investments with and without volatility exposure enables an answer to the research question about an improvement of the results by the addition of a non-standard asset to the diversified portfolios. The VXX is considered as the proxy for volatility as it is the most popular ETN which follows the volatility index derivatives with the given maturity. To test the robustness of the results the portfolios are constructed with a broad range of different parameters and assumptions imposed on the optimization procedure.

**Keywords**: volatility, asset class, portfolio optimization, Polish market, VIX, Markowitz portfolio, naïve diversification.

**JEL codes**: C4, C45, C61, C15, G14, G17.

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## Introduction

Volatility as an asset class is a relatively new concept in financial theory but due to its rapid development and new investment possibilities the interest of academia and professionals has increased remarkably. In previous years, volatility has been considered as one of the features of investment which corresponds with the risk related to the uncertain outcomes. An introduction of the VIX index in 1993 by the Chicago Board Options Exchange whose aim was to measure the market's expectations of volatility implied from the options' quotations, and the launch of derivatives based on the VIX eleven years later, are the key dates for the new investment possibilities. Even though the VIX methodology has changed during the last 25 years its core characteristics remain the same—the turbulences and concerns about markets' dynamics were followed by the high values of the volatility index (VIX is commonly known as a "fear index"). A negative correlation with most markets attracted many investors who were interested in non-standard assets.

The VIX index is not directly investable and the investor who wants to obtain the exposure to volatility needs to use derivatives. Although it is technically possible to directly replicate the VIX—it requires an extremely sophisticated strategy that usually involves high transactional costs. The figures confirm a great increase in the interest of volatility derivatives since their introduction. According to the CBOE (2018) data, the average daily turnover of VIX futures has grown from 462 in 2004 to almost 300,000 in 2018 (with CAGR higher than 58%). A similar pattern was observed in the case of VIX options: a daily volume of traded contracts has risen from 23,500 in 2006 up to over 650,000 in 2018 (with CAGR on the level of 32%). As the American volatility index has been gaining popularity, other stock exchanges have introduced similar products like the VSTOXX index (on Eurex), Nikkei Stock Average Volatility Index (in Japan) or the VDAX index (in Germany).

The popularity of volatility indices and their derivatives has brought a broad range of investment opportunities to exploit. The popular examples, which commonly appear in the literature, are the following:

– a volatility risk premium (VRP) which benefits from the historically persisting difference between implied and realized volatility;
– trading on VIX futures term structure and seeking arbitrage possibilities;
– portfolio diversification and portfolio hedging which is possible thanks to the negative correlation of the volatility index with the market, especially during times of turbulence (conditional correlation).

The paper's aim is to focus on portfolio diversification and the benefits associated with it.

The studies of American and European markets are the popular goal for investigations. The development and progress of volatility instruments have encouraged the researchers to explore the subject of this new asset class. A broad

range of the academic papers, which examine the investment possibilities with the VIX, VSTOXX or other volatility indices, has been already widely documented. Due to their non-standard characteristics of negative correlation with the market, trading volatility is a good option for extra profits and an effective way of hedging or diversifying the portfolio. The main goal of this paper is to analyse the diversification opportunities using volatility instruments from the Polish market perspective. The simulation is devoted to the hypothetical investor who operates in Poland and has a diversified portfolio of Polish assets but at the same time has also access to global markets. The research seems to be interesting as most of the papers in the literature focus on American and European markets only.

The analysis focuses on diversification benefits of adding volatility exposure to the portfolio containing a wide range of different assets. The research tries to answer the main question: *Whether a hypothetical investor that has a diversified portfolio can improve his efficiency, measured by the risk-adjusted return, by an inclusion of volatility exposure?*

The research presents a comparison of two types of portfolios—the first one containing volatility exposure and the second one with the standard type of assets only. To test the hypothesis about the beneficial effect of the volatility exposure to the diversified portfolio the following research questions were formulated:

– What are the results of the Markowitz portfolio optimization?
– Does the naïve diversification method perform better or worse than the Markowitz?
– What is the sensitivity of the results to the chosen parameters?

The answers to the above questions may lead to a rejection or a confirmation of the hypothesis.

The Warsaw Stock Exchange (WSE) has a relatively short history and the Polish financial market is still underdeveloped compared to western European countries which results in the illiquid derivatives market and no volatility instruments at all. Therefore, the abovementioned investor cannot invest in volatility implied from the WIG20 option and cannot have direct exposure to Polish volatility. However, Ślepaczuk and Zakrzewski (2008) in their paper have proposed the VWIG20—volatility index for the WIG20 index which is equivalent to VIX, VSTOXX and other volatility indices. They presented that an introduction of such an index would be a beneficial improvement for the Polish financial market and would boost the derivatives sector and provide new investment alternatives. The VIX, as the most popular volatility index that has the broadest offer of derivatives and ETFs, is an intuitive solution to overcome a lack of the Polish alternatives. On the one hand, the standard asset classes, such as equity or bonds, exhibit common characteristics and move in the same direction, no matter where they are geographically located. On the other hand, the Polish market may be vulnerable to the shocks that are not observable on

the American market and the VIX inclusion will not diversify the portfolio efficiently. The goal of this research is to look closer into this matter. The VXX, an Exchange Traded Note which provides exposure to the short-term VIX futures, will serve as a proxy of volatility in this analysis. Thanks to its high liquidity and long history of quotations, it allows for robust results.

The paper has the following structure. Section 1 is dedicated to the literature review. The next chapter describes the methodology, i.e. what kind of methods were performed, which parameters were used and what assumptions were established. Section 3 contains the description of data: what kind of assets were used, their characteristics, features, and summary. The following chapter shows the results of the research and the sensitivity analysis is presented to check the robustness of the outcomes. The last section summarizes the research, draws conclusions and presents a way for further development and improvement.

## 1. Literature review

Volatility commonly refers to the risk associated with the investments in the financial markets. The uncertain outcomes lead to many risk management solutions and applications. Correct measurement of volatility and ability to forecast its future value are important parts of the business for banks, insurance companies, investment funds, and other financial institutions. Academia followed this tendency providing many types of research devoted to risk management and volatility forecasting.

Introduction of the VIX futures, options, and other derivatives allowed the investors to look on volatility from a different perspective—as a new asset class (Jabłecki, Kokoszczyński, Sakowski, Ślepaczuk i Wójcik, 2015). The academic papers dedicated to the investment possibilities and portfolio diversification with volatility instruments are not as popular as the previous subject, but this stream is still developing as academia provides new research ideas and results. This section presents empirical work contributing to this matter.

Szado (2009) focused his analysis on the diversification impact of VIX derivatives during the last financial crisis in 2008 and its long-term benefits. He divided his research into two periods: the first one begins with an introduction of the VIX options in March 2006 and finishes at the end of 2008; the second one is particularly devoted to the last financial crisis and consists of the August--December 2008 period. He established different portfolio types and added volatility exposure using VIX options and futures to them. Once the portfolio was constructed no rebalancing was done until the end of the analysis period. The comparison of different portfolios led to the conclusion that the VIX was an effective hedging instrument during the market turmoil of 2008. The long-term relationship is, however, less efficient and provided poor investment results due to the negative expected return of the VIX derivatives.

Briere, Fermanian, Malongo and Signori (2010) focused their analysis on European markets and possible ways of hedging portfolios through volatility instruments. They analysed both global and European equity in individual countries. The authors performed portfolio optimization and compared the results of adding long volatility exposure using VSTOXX and VIX futures. The research covered the period from 1999 to 2010. As the futures data are available only since 2004 for the VIX and 2009 for the VSTOXX, the authors estimated futures price by the average linear relationship between the spot price of the volatility indices and derivatives. The authors concluded that the incorporation of the volatility exposure to the European indices had a positive impact on their performance and was an effective way of hedging. The effect was stronger for the VSTOXX futures than the VIX derivatives as the European volatility index futures better covered the risk associated with the eurozone equity downturns than American ones. In addition, the authors suggested that the steeper VIX futures term structure incurs higher costs of carry than the VSTOXX.

Alexander and Korovilas (2011) focused their analysis on difficulties related to portfolio diversification with volatility instruments caused by the high rolling costs of futures contracts. The research was conducted on SPY (an ETF on S&P 500) and 3 maturities of the VIX futures contracts. To reduce the negative impact of high volatility when the contracts are close to the expiration, the rollover is done 5 days before the maturity of the VIX futures. The research was done by means of the Markowitz framework and its extension—the Black-Littermann. Initially, the authors were analysing portfolio diversification with VIX futures *ex-post*. They used the mean-variance framework and its generalised Sharpe Ratio (GSR) extension. The results confirmed the usefulness of volatility exposure but mainly during market turbulence. In the *ex-ante* analysis of the Markowitz framework, the authors emphasized the dependency of results on the in-sample data. However, the conclusions remain similar as VIX futures perform well during the large drops. The investor's views in the Black-Littermann model were presented in two ways. The results confirm that the VIX exposure deteriorates returns in the long-term. The authors concluded that volatility instruments should be left for the speculative purpose rather than used by long-term investors, like mutual and pension funds.

In the following paper, Alexander and Korovilas (2012) made extensive research about the Exchange Traded Notes (ETN) on VIX futures. A high cost of carry, related to rolling over the futures contract, significantly decreases the performance of the ETNs based on the VIX, e.g. VXX, which is the most popular product within the class. A difference between the investment results of the VXX (ETN on short-term contracts) and VXZ (ETN on mid-term VIX futures) emphasizes the importance of rolling costs. To limit their negative impact the authors analysed the investment results of the XVIX and XVZ ETNs which are the combination of the mid-term VIX futures (VXZ) and the inverse of the short-term contracts (VXX). The authors noticed that the XVIX performs

well when the market is calm and the term structure of VIX futures is in con-tango, while the XVZ achieves better results during the market's downturn. It led them to exploit the volatility instruments' strategy even more and use the mix of both the abovementioned ETNs. A general conclusion of the investment results is that the presented strategy may achieve very good results, perform well in every market condition and be an interesting alternative for a long-term investor who seeks an alternative solution to its portfolio.

Stanescu and Tunaru (2013) added volatility exposure to portfolios from American and European markets. They used the S&P500 and Euro Stoxx 50 as the proxies for equity markets and the Barclays Aggregated bond index which represents the debt instruments. They constructed equity only and equity-bond portfolios and added volatility index futures in a similar way as Szado (2009). The results of the analysis proved the authors' hypothesis that adding volatility instruments to the portfolio produces not only effective hedging but also contributes to the higher risk-adjusted return, especially during the mar-kets' downturns. The authors also performed a statistical arbitrage strategy on the VIX-VSTOXX futures differences. The results showed that statistical ar-bitrage on the volatility index futures' difference may provide profits over the period of analysis.

Jabłecki and others (2015) analysed the impact of VIX futures on the port-folio diversification using the Markowitz framework and its extension: Black-Littermann model. The analysis covered the period from 2006 to 2013. The first part focused on the portfolio which consisted of the American equity, global bonds and commodity instruments. 60, 120, 180 and 240 trading days were selected as a *memory* parameter on which weights were calculated. The constructed portfolios were invested for 1, 2, 4 and 8 weeks and after that pe-riod the portfolio was established again with different weights. The authors also included the possibility of short selling the assets. The results depended on the selected parameters: long investment horizon (8 weeks) and long memory (240 trading days) produced poorer results. However, the possibility of a short selling generated higher profits for portfolios with the VIX. Analysing the re-sults year by year led to the conclusion that VIX has a beneficial effect during market downturns for all parameters, whereas the performance during other periods depends on the selected criteria.

The second part was devoted to the Black-Littermann model. This part con-tained ten different equity indices from developed and emerging countries. Rebalancing was done every week and the last 13, 26 and 52 weeks were con-sidered as a *memory* parameter. The general conclusions were again ambigu-ous and considerably depended on the selected scenario. VIX futures inclusion, however, had a positive effect during the financial crisis (2008) and markets' decrease (2011) which confirmed the findings of the mean-variance analysis. These results suggested that the volatility index futures may be a powerful risk management tool but it requires the attentive selection and proper research.

Caloiero and Guidolin (2017) optimized portfolios consisting of American assets and volatility exposure. Instead of VIX futures or options the authors used Exchange Traded Products and checked how their performance affected investment profitability. The authors performed the optimization using three different types of investors' utility function—mean-variance, power and negative exponential utility function. In addition, they compared the results between the portfolios without volatility exposure, with the VIX index, and with the VXX ETN as one of the most popular volatility Exchange Traded Product. The analysis used weekly data, the long-only portfolio was constructed and the time period ranged between January 2009 and February 2016. The authors showed that the VIX index lies on the efficient frontier and improves the investment results when assessing them *ex-post*. Nonetheless, the VXX remains an important part of the portfolio only for special preferences and assumptions of an investor. The results of the simulated investment strategies presented similar conclusions.

On the other hand, there is a set of papers analysing the efficiency of long/short volatility strategies when they are one of the main constituents of the portfolio. Dondoni, Montagna and Maggi (2018) presented the results of different portfolio strategies extending a long / short position on VIX Futures. Fahling, Steurer, Schädler and Volz (2018) showed how VIX futures and options can hedge equity portfolios and when they are superior to traditional hedging alternatives and compares the outcome of a VIX hedging strategy with a Buy & Hold strategy of the S&P 500 index over a time period of 20 years. Ślepaczuk and Zakrzewski (2013) analysed properties of various volatility estimators which can be used in the process of long/short volatility signals generation. Szado (2019) found that meaningful portfolio return enhancements are possible over particular time periods with small allocations to VIX futures selling.

## 2. Methodology

The main goal of the analysis was to compare the performance of the portfolios with and without any volatility exposure. An Information Ratio, which is a ratio of annual compounded return to the annual standard deviation of daily returns, was chosen as a decisive criterion.

$$ARC(\text{Annual Return Compounded}) = \left( \prod_{t=1}^{N} \left( 1 + R_t \right) \right)^{252/N} - 1, \qquad (1)$$

where $R_t$ is the percentage daily return on day $t$, defined as: $R_t$.

$$ASD \ (\text{Annual Standard Deviation}) = \sqrt{\frac{252}{N-1} \sum_{t=1}^{N} (R_t - \overline{R})^2}, \qquad (2)$$

where $\bar{R}$ is the average of all *n*-returns.

$$\text{IR1 (Information Ratio)} = \frac{ARC}{ASD}, \qquad (3)$$

A higher value of the IR of the portfolio which includes the VXX (an ETF on short-term VIX futures) would lead to the conclusion that adding volatility exposure improves the efficiency of the investments thanks to the diversification benefits and vice versa. Risk-adjusted measures are the popular indicators of investment profitability and intuitive comparison could be made between portfolios with different parameters. In addition, to reflect better the assets' behaviour during the period of analysis, comparisons of maximum drawdowns and an information ratio weighted by maximum drawdown were done for descriptive statistics purposes (detailed values are presented in section 4).

$$\text{MD (Maximum Drawdown)} = \max_{\tau\varepsilon[0,\,T]} (\max_{\tau\varepsilon[0,\,T]} R_t - R_\tau) \qquad (4)$$

$$\text{Information Ratio 2 (IR2)} = \text{sign}(ARC) * \frac{ARC^2}{(ASD * MD)} \qquad (5)$$

A more detailed analysis of the consequence of the usage of Sharpe or Information ratio, especially during the period of negative (excess) returns, was presented in Israelsen (2005). The research methodology is divided into two parts. The first one uses the breakthrough Markowitz framework to calculate the efficient frontier. Next, the efficient frontier is scanned in order to find the point with the highest return to risk ratio, where risk is associated with a standard deviation. Finally, the shares of each asset are extracted from the chosen point on the efficient frontier and investment is done according to the calculated weights.

The mean-variance analysis is performed on the past returns of assets. Historical returns and a variance-covariance matrix determine the weights of the portfolio. A length of a chosen period is a key parameter for the estimated results and could affect the outcomes—this parameter is called *memory* for the purpose of the analysis. Historical data from the last 15, 30, 60 and 120 trading days were selected for each case. The length of the investment depends on the value of another parameter called *rebalancing*. It determines how often weights are calculated and new portfolios are constructed. The research considers three frequencies of rebalancing: weekly, monthly and quarterly. After the end of each period, the Markowitz analysis is performed one more time and the investment with a different share of assets is computed again.

The first part of the research is performed on four different scenarios each assuming different weight constraints imposed on the portfolio. The basic one considers the "long only" portfolio—weights are restricted from 0% to 100%

interval. To reduce the impact of one asset on the whole portfolio performance, the second scenario assumes the limitations of the upper bound to 25%. Such an approach may be useful when historical data are not good predictors for future performance which results in a too large share of one asset within the portfolio. The third scenario allows for the short selling, i.e. the situation when weights may have negative values and the investor could benefit from decreasing the value of the asset. The weight interval for this case ranges from –100% to 100%. Similarly, to the second case, the weights in the last scenario are restricted from –25% to 25%. A sum of every asset share equals to 100% in all cases. Such a broad range of parameters and different scenarios allows for robust sensitivity analysis and durable conclusions.

The Markowitz portfolio has laid the foundations for the modern financial theory and serves as a benchmark in many studies. The limitations of this framework have been widely documented, e.g. unrealistic assumptions about the market microstructure (e.g. no taxes or transactional costs) or considering only past performance and assuming a normal distribution of the assets' returns. The aim of the second part of the research is not to overcome these disadvantages but rather to present another way of investing. The methodology is similar to that used by Szado (2009) where he applied a naïve diversification. That approach suggests imposing a fixed value on weights in the portfolio at the beginning of each period. At the end of each period the weights return to the previous values as established before. Similarly, to the previous part, rebalancing is done on a weekly, monthly and quarterly basis.

Even though the second approach seems to be less sophisticated—it is a popular concept of investing in a mutual or retirement funds. What is more DeMiguel, Garlapii and Uppal (2007) showed that the naïve diversification approach does not systematically underperform when compared to the mean-variance framework. The authors concluded that more attention should be paid to the estimation of assets' moments and thus reducing the estimation errors which is not the case for the naïve approach. In addition, naïve diversification reduces the transaction costs, limits the extreme position of one asset and is a sensible choice for the investors who are oriented towards passive investments and prefer to follow the market trend. What is more Benartzi and Thaler (2001) showed that most individuals follow the simple rule of thumb and evenly distribute welfare across different asset classes. Although such a framework may not be the most effective the comparison of results of the naïve diversification with the Markowitz approach may lead to different conclusions.

To assess the effectiveness of adding the volatility exposure to a diversified portfolio the naïve approach compares the risk-adjusted return between the portfolios with 1%, 3% and 5% shares of the VXX and without volatility instrument at all. Based on this assumption, the following four types of scenarios were tested, each with different portfolios' characteristics. The first one represents the risk-averse approach to investing—with high shares of debt and

**Table 1. Asset shares in naïve diversification strategy**

| Scenario | WIG20 | UniKorona | NN Obligacji | S&P 500 | MSCI Europe | FTSE Pacific | FTSE G7 Bond | S&P GSCI | DJ Real Estate | VXX |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | (%) | | | | | |
| First | 20 | 40 | 20 | 2 | 2 | 2 | 10 | 2 | 2 | 0 |
| | 20 | 40 | 20 | 1.8 | 1.8 | 1.8 | 10 | 1.8 | 1.8 | 1 |
| | 20 | 40 | 20 | 1.5 | 1.5 | 1.5 | 9.5 | 1.5 | 1.5 | 3 |
| | 20 | 40 | 20 | 1.2 | 1.2 | 1.2 | 9 | 1.2 | 1.2 | 5 |
| Second | 40 | 20 | 20 | 4 | 4 | 4 | 4 | 2 | 2 | 0 |
| | 40 | 20 | 20 | 3.8 | 3.8 | 3.8 | 3.8 | 1.9 | 1.9 | 1 |
| | 40 | 20 | 20 | 3.5 | 3.5 | 3.5 | 3.5 | 1.5 | 1.5 | 3 |
| | 40 | 20 | 20 | 3 | 3 | 3 | 3 | 1.5 | 1.5 | 5 |
| Third | 70 | | | 10 | 10 | 10 | | | | 0 |
| | 70 | | | 9.5 | 9.75 | 9.75 | | | | 1 |
| | 70 | | | 9 | 9 | 9 | | | | 3 |
| | 70 | | | 8 | 8.5 | 8.5 | | | | 5 |
| Fourth | 40 | | | 20 | 20 | 20 | | | | 0 |
| | 40 | | | 19.5 | 19.75 | 19.75 | | | | 1 |
| | 40 | | | 19 | 19 | 19 | | | | 3 |
| | 40 | | | 18 | 18.5 | 18.5 | | | | 5 |

Note: *Scenario* parameter indicates a type of constructed portfolio. Detailed information is presented in the Methodology section.

Source: All data presented in Table 1 and in the next tables are authors' calculations.

money market instruments. The second scenario is dedicated to a more aggressive strategy with a relatively higher value of equity and having the same type of assets as in the first scenario (the detailed description of components used is presented in the section Data). Both portfolios have a stable ratio between Polish and global assets: 80% and 20%, respectively. The third and the fourth type of portfolios simulate the investment fund which is devoted to the only-equity strategy. These results may differ from the two previous scenarios as the stocks are more strongly correlated with the volatility instrument. The third scenario exhibits higher exposure to the Polish assets, while the fourth is more geographically diversified: the WIG20 share is equal to 70% and 40%, respectively. The added exposure of the VXX in each scenario is distributed in such a way that the ratio of Polish and global assets remains stable. Table 1 presents the weights for each portfolio while Table 2 summarizes the assumptions about the tested scenarios.

**Table 2. Summary of the assumptions for tested scenarios**

| Methodology | Scenario | Assumptions |
|---|---|---|
| Markowtiz | first | a long only strategy: the weight of single assets may vary between (0% : 100%) |
| | second | a long only strategy with an upper bound limitation to 25%: (0% : 25%) |
| | third | an allowance for short selling: the weights are restricted to (−100% : 100%) range |
| | fourth | an allowance for short selling with an upper and lower bound limitation (−25% : 25%) |
| Naïve | first | a diversified portfolio with high share of safe assets, i.e. money market and debt instruments |
| | second | a diversified portfolio with relatively higher share of more aggressive assets, i.e. equity |
| | third | an equity only portfolio with 70% of shares dedicated to the polish market |
| | fourth | an equity only portfolio which is more geographically diversified (40% for the polish equity and 60% for global indices) |

Note: The table presents the assumptions of the tested scenarios for each methodology.

All calculations were performed in the local currencies. During the period of analysis the interest rates in Poland were higher than in the United States and Europe because of the central bank's policies and their Quantitative Easing programme (except the negligible short period in 2018 when FED's reference rate rose above the NBP's interest rate). Thanks to a positive difference between the Polish and European and American interest rates, that was indicated in

Figure 1, the beneficial hedge could be made with a forward instrument. The forward exchange rate of Polish zloty to euro or USD was lower than the spot price which made it possible not to incur additional costs related to hedging of exchange rates.
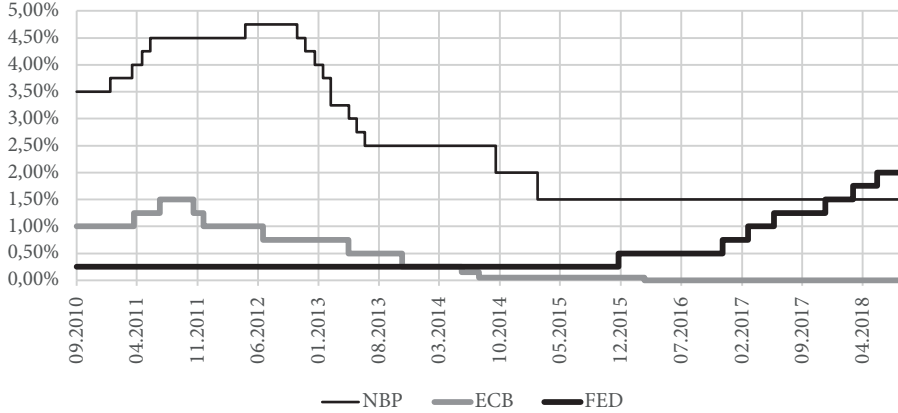


**Figure 1. The plot of interest rates of NBP, ECB and FED**
Note: The plot presents the changes of reference interest rates of Polish, European and American central banks. The figures cover the period from September 2010 to August 2018. Source: (https://www.federalreserve.gov, https://www.ecb.europa.eu, https://www.nbp.pl).

Transaction costs ($k_t$ = 0.25%) are included for all scenarios to mirror the reality of the investments. The portfolio is charged with the transaction costs every time the portfolio is rebalanced, launched and liquidated. To include the costs into the portfolio performance the Turnover Ratio is multiplied by the value of transaction costs and then subtracted from the daily return. The Turnover Ratio shows how much the portfolio shares changed. This value varies between 0% (the weights remained the same) to twice the range of possible asset shares (which is a total reallocation of the assets). For example, the Turnover Ratio for a long-only portfolio varies between 0% and 200%.

$$\text{Turnover Ratio (TR)} = \sum_{i=1}^{N} \left| w_{i,t} - w_{i,t-1} \right|, \qquad (6)$$

where $w_{i,t}$ is a weight of ith asset on a $t$-day.[4]

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[4] All calculations were performed in an R Studio environment.

## 3. Data

The analysis is performed on the daily close prices and covers the period since the end of September 2010 when an ETF tracking the WIG20 index was introduced, to July 2018. All the constituents presented below are either the ETFs or mutual investment funds' quotations which allows for easy calculation of all capital gains and management fees. Besides, it mirrors the real-world investments and makes the analysis robust. The data were downloaded from stooq. com, yahoo!finance and the ETFs' providers webpages. The following ten types of assets were selected to represent the diversified portfolio:

– WIG20 index, represented by Lyxor WIG20 UCITS ETF.
– UniKorona Dochodowy, representing the Polish money market.
– NN Obligacji, a fund that serves as a proxy for the Polish treasury market.
– S&P500 index, represented by SPDR S&P500 ETF.
– MSCI Europe index, covering the behaviour of stocks from 15 developed countries in Europe. The ETF is called iShares Core MSCI Europe and is run by BlackRock.
– FTSE Pacific, an index of equities located in the Pacific area (i.e. Japan, Australia, Hong Kong, South Korea and Singapore). The ETF is managed by Vanguard.
– FTSE G7 Government Bond, an index that measures the performance of sovereign bonds of G7 countries. It is represented by iShares Global Govt Bonds UCITS ETF.
– S&P GSCI, an index that serves as a benchmark for the commodity markets. iShares S&P GSCI Commodity-Indexed Trust tries to mimic the index behaviour.
– Dow Jones Real Estate, an index that represents the performance of REIT and the other companies related to a real estate investments. An ETF is provided by SPDR.
– VXX, an ETN which provides the exposure to short-term VIX futures.

Table 3 presents a correlation matrix of the assets used in the analysis. VXX has a strong negative correlation with foreign equities (S&P500, MSCI Europe and FTSE Pacific), commodity and real estate components as was expected. Polish assets are less prone to movements of the VXX as correlation with the NN Obligacji fund equals to –0.14, the money-market fund has no common co-movement with volatility, while the WIG20 index's correlation with the VXX amounts to –0.32. The above observations may be an initial suggestion for the diversification benefits of adding volatility exposure to the portfolio.

Table 4 displays a summary of the asset's performance and basic descriptive statistics during the period of analysis while Figures 2 and 3 display the equity lines of each asset.

Considering IR1 UniKorona Dochodowy exhibits the best result which amounts to 2.54. Although its return is not large an extremely small value of

**Table 3. Correlation matrix**

| Correlation matrix | WIG20 | UniKorona Dochodowy | NN Obligacji | S&P 500 | MSCI Europe | FTSE Pacific | FTSE G7 Government Bond | S&P GSCI | Dow Jones Real Estate | VXX |
|---|---|---|---|---|---|---|---|---|---|---|
| WIG20 | 1 | | | | | | | | | |
| UniKorona Dochodowy | 0.0249 | 1 | | | | | | | | |
| NN Obligacji | 0.1704 | 0.1326 | 1 | | | | | | | |
| S&P 500 | **0.3821** | 0.0013 | 0.1264 | 1 | | | | | | |
| MSCI Europe | **0.5188** | 0.0171 | 0.2054 | **0.6201** | 1 | | | | | |
| FTSE Pacific | **0.3710** | 0.0010 | 0.1655 | **0.8245** | **0.6062** | 1 | | | | |
| FTSE G7 Government Bond | –0.0519 | 0.0592 | 0.2185 | –0.1359 | –0.2302 | –0.0637 | 1 | | | |
| S&P GSCI | 0.2448 | –0.0087 | 0.0813 | **0.4211** | **0.3320** | **0.4192** | –0.0190 | 1 | | |
| Dow Jones Real Estate | 0.3253 | 0.0210 | 0.2400 | **0.8132** | **0.5204** | **0.7745** | 0.0473 | **0.3495** | 1 | |
| VXX | –0.3200 | 0.0051 | –0.1430 | **–0.8297** | **–0.5119** | **–0.7020** | 0.1078 | –0.3201 | **–0.6613** | 1 |

Note: The table presents the correlation matrix for the assets used in the research. The calculation was prepared from September 2010 to July 2018 for the simple daily returns. Bolded observations indicate semi-strong and strong correlations: $|\rho| \geq 0.33$.

**Table 4. Assets performance and descriptive statistics**

| | WIG20 | UniKorona Dochodowy | NN Obligacji | S&P 500 | MSCI Europe | FTSE Pacific | FTSE G7 Government Bond | S&P GSCI |
|---|---|---|---|---|---|---|---|---|
| ARC | 1.31% | 3.64% | 4.15% | 14.08% | 7.77% | 6.7% | 0.47% | -6.19% |
| ASD | 18.83% | 1.43% | 2.76% | 13.94% | 15.85% | 15.93% | 6.3% | 18.52% |
| IR1 | 0.07 | 2.54 | 1.5 | 1.01 | 0.49 | 0.42 | 0.07 | -0.33 |
| MD | 32.47% | 2.58% | 5.44% | 18.61% | 25.9% | 23.23% | 13.02% | 68.78% |
| IR/MD | 0.22 | 98.45 | 27.64 | 5.43 | 1.89 | 1.81 | 0.57 | -0.49 |
| IR2 | 0 | 3.58 | 1.15 | 0.76 | 0.15 | 0.12 | 0 | -0.03 |
| Skewness | -0.19 | -19.18 | -0.5 | -0.51 | -0.34 | -0.43 | -0.16 | -0.18 |
| Kurtosis | 4.54 | 477.71 | 4.72 | 5.12 | 3.68 | 4.13 | 1.58 | 2.1 |
| Dow Jones Real Estate | 0.3253 | 0.0210 | 0.2400 | 0.8132 | 0.5204 | 0.7745 | 0.0473 | 0.3495 |
| VXX | -0.3200 | 0.0051 | -0.1430 | -0.8297 | -0.5119 | -0.7020 | 0.1078 | -0.3201 |

Note: ARC—annualized return compounded, ASD—annualized standard deviation, IR1 and IR2— Information Ratios, presented in section 3, MD—maximum drawdown. All these statistics were prepared from September 2010 to July 2018 for the simple daily returns.
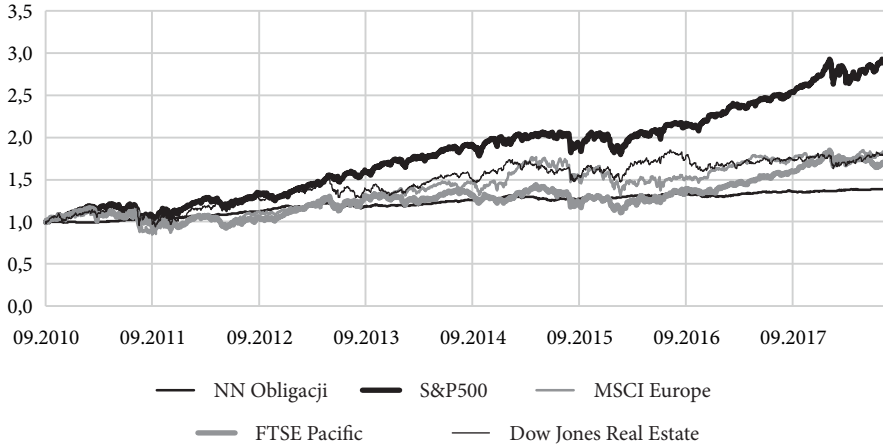
**Figure 2. The equity line for 5 assets with the highest returns**
Note: The plot presents the equity line for 5 assets with the highest returns. The figures were
prepared from 2010 to July 2018 for the simple daily returns.
Source: Own elaboration.
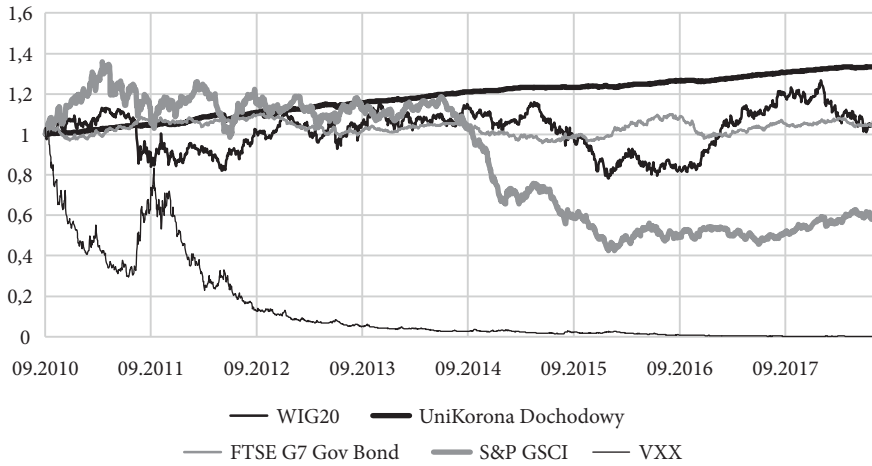


**Figure 3. The equity line for 5 assets with the lowest returns**
Note: The plot presents the equity line for 5 assets with the lowest returns. The figures were
prepared from September 2010 to July 2018 for the simple daily returns.
Source: Own elaboration.

the standard deviation contributes to the high IR1. The second best result is
obtained by the NN Obligacji fund which IR equals to 1.5 while the return is
subtly higher than the money market fund the risk factor has almost doubled.
S&P500 has the highest ARC over the analysed period (approximately 14%) and
a relatively good risk-adjusted return which equals to 1. Real estate, European
and Pacific equities have performed decently—their IR1s vary around 0.45,
whereas global bonds and Polish equities ARC are close to 0. A serious de-

crease in oil prices has resulted in poor performance of the S&P GSCI while the VXX has lost almost all its value. The reason for that situation is the fact that the ETN was launched at the beginning of 2009 when the financial crisis hit the markets and high values of the VIX were observed. Then, the volatility dropped which brought considerable decreases of the VXX quotations. Most of the assets are negatively skewed, whereas values of the kurtosis vary.

# 4. Results

The results presented in this section provide a deep analysis of the portfolios' performance. The broad range of parameters and assumptions contribute to many alternative variants. The annual return compounded (ARC), the annualized Information Ratio, the average of transaction costs and the average of assets weights are calculated for each portfolio. All presented figures below are net values.

## 4.1. Markowitz portfolio

Tables 5 and 6 display the results of ARC and IR for all selected assumptions and specifications. The results of the first scenario are unambiguous. The portfolios without VXX have the highest values of the risk-adjusted returns for the memory of 120 days and weekly and monthly rebalancing. On the other hand, when comparing the portfolios which included volatility exposure, quarterly reallocation and short memory tend to perform better than other parameters. Nevertheless, the inclusion of VXX into the optimization process does not produce better results of the Information Ratio in any case.

   The average of the asset shares[5] for the first scenario for a portfolio without and with VXX indicates that both types of portfolios are dominated by the money market fund where average weights vary between 62% and 80%. NN Obligacji is the second most often chosen asset within the portfolios where the average weight amounts to approximately 10%–12%. Remaining assets have a much smaller impact on the portfolio performance. An important observation is that the shorter the *memory* parameter and the less often the frequency of rebalancing, then the portfolio is less dominated by the UniKorona Dochodowy. It could be explained by more stable weights across different periods when longer historical data are taken into consideration and the impact of any extreme events does not influence the results so heavily. The average share of the VXX varies between 1% and 4%: it is higher for the quarterly rebalancing. An inclusion of volatility exposure to the portfolio results in a bigger share of the S&P500 and smaller of the global government bonds which is quite intuitive.

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   [5] The tables with the average asset share for the each scenario for portfolio without and with the VXX can be obtained on request.

**Table 5. Annual return compounded (ARC) of the Markowitz portfolio with and without the VXX exposure**

| Rebalancing on | Memory | 1st scenario (%) | | 2nd scenario (%) | | 3rd scenario (%) | | 4th scenario (%) | |
|---|---|---|---|---|---|---|---|---|---|
| | | no VXX | with VXX | no VXX | with VXX | no VXX | with VXX | no VXX | with VXX |
| Weeks | 15 | -3.23 | -6.27 | -2.27 | -6.73 | -13.39 | -17.82 | -6.62 | -7.63 |
| | 30 | -0.57 | -5.28 | -1.44 | -4.22 | -3.19 | -5.8 | -4.06 | -4.89 |
| | 60 | 1.79 | 1.05 | 1.00 | 0.47 | -0.34 | -2.67 | 0.55 | **1.67** |
| | 120 | 3.48 | 0.68 | 1.89 | 0.28 | 2.41 | 1.55 | 0.27 | **1.26** |
| Months | 15 | 1.27 | -4.15 | 2.28 | -1.1 | -0.9 | -2.16 | -0.92 | -1.18 |
| | 30 | 1.92 | -2.16 | 2.18 | -0.79 | 0.85 | 0.2 | 0.84 | 0.84 |
| | 60 | 1.91 | 0.94 | 3.86 | 2.43 | 2.64 | 2.33 | 3.51 | **4.8** |
| | 120 | 3.86 | -0.57 | 2.96 | 1.29 | 2.94 | 3.54 | 2.25 | **3.04** |
| Quarters | 15 | 4.67 | 1.71 | 3.27 | 2.17 | 2.4 | 0.63 | 2.14 | 1.14 |
| | 30 | 3.71 | 2.21 | 3.1 | 2.49 | 2.56 | 1.43 | 1.44 | 1.14 |
| | 60 | 0.9 | 0.42 | 3.35 | 1.28 | 2.65 | **3.36** | 4.17 | 2.33 |
| | 120 | 2.97 | -0.47 | 2.66 | 0.32 | 3.12 | **3.51** | 2.77 | **2.8** |

Note: The table presents the ARC during the period of analysis. The 1st scenario assumes that one asset share may range between 0% and 100%, the 2nd scenario limits one asset weight upper-bound to 25%, the 3rd scenario allows for short-selling and relax the previous limitation (a single asset shares vary between –100% and 100% and the 4th scenario limits lower and upper bound to –25% and 25%, respectively. *Memory* parameter shows how many historical trading days were used to establish an efficient frontier, whereas *Rebalancing on* parameter presents the frequency of the portfolio reallocation. The bold numbers indicate portfolios which include the VXX and have higher values of ARC.

The second scenario which assumes a limitation of upper bound share to 25% leads to a similar conclusion: all parameters provide worse results for the portfolio with the VXX than without in terms of returns and IR. The portfolios which include volatility ETN follow the same pattern as in the first scenario. In comparison to the previous scenario the portfolios with the VXX and upper bound limitation improve results for monthly and quarterly rebalancing, whereas the difference of first and second scenarios for portfolios without volatility exposure does not follow any pattern. Nevertheless, the general conclusion remains the same that adding volatility does not enhance the overall performance.

Average asset weights for the second scenario for portfolio without and with VXX indicates that an upper bound limitation of a single asset share provides more diversified portfolios but UniKorona Dochodowy remains the most important component with an average weight equalled to over 24%. Similarly, to the previous scenario, the NN Obligacji, S&P500 and FTSE G7 Government Bond index play an important role within the portfolio performance. The VXX share slightly increased and it caused an increase of American equity and a decrease of the global bonds. Contrary to the previous scenario, the asset weights are more stable across different *memory* or *rebalancing* parameters.

The third scenario allows for short-selling and the weight of each asset may vary from –100% to 100%. The ARC of portfolio with the VXX presents better results than without volatility exposure in 2 out of 12 cases (for quarterly rebalancing and the memory of 60 and 120 trading days), whereas the Information Ratio is higher only in 1 case which is uninterpretable due to its negative value. Both types of portfolios exhibit two similar patterns: longer memory and less frequent rebalancing provide better results in terms of absolute returns and the Information Ratio. The portfolios in the third scenario are even more dominated by the UniKorona Dochodowy than in the first as its weights vary around 82%. Remaining assets follow a similar order as previously—NN Obligacji, the American equity and global bonds have the greatest weights around the leftover components. The VXX share decreases in comparison to the first two scenarios and ranges between 0% and 1%.

The results for the last scenario much differ from the previous cases. The portfolios with the VXX have higher Information Ratio in 6 out of 12 situations, mostly when the *memory* parameter equals to 60 and 120 trading days. Besides, volatility exposure improves the ARC of the portfolio in 5 out of 12 cases. The average weights for the fourth scenario are positive in almost all cases. The general order is followed from the previous scenarios: the UniKorona Dochodowy, NN Obligacji, S&P500 and FTSE G7 government bonds indices have the highest shares across the portfolios. The share of the VXX varies around 1.5%–3% and is considerably higher for a shorter memory.

An analysis of the portfolio performance constructed using the Markowitz framework exhibits a clear pattern—weekly rebalancing produces poor invest-

**Table 6. Annualized Information Ratio of the Markowitz portfolio with and without the VXX exposure**

| Rebalancing on | Memory | 1st scenario | | 2nd scenario | | 3rd scenario | | 4th scenario | |
|---|---|---|---|---|---|---|---|---|---|
| | | no VXX | with VXX | no VXX | with VXX | no VXX | with VXX | no VXX | with VXX |
| Weeks | 15 | −0.6919 | −0.9116 | −0.4500 | −1.2052 | −2.5005 | **−2.3043** | −1.0876 | −1.2250 |
| | 30 | 0.1875 | −0.7269 | −0.2889 | −0.7994 | −1.0675 | −1.3716 | −0.7098 | −0.8489 |
| | 60 | 0.5437 | 0.2220 | 0.2070 | 0.1070 | −0.1343 | −0.7010 | 0.1017 | **0.3364** |
| | 120 | 2.1868 | 0.1120 | 0.3826 | 0.0558 | 1.4066 | 0.7054 | 0.0494 | **0.2447** |
| Months | 15 | 0.2520 | −0.6712 | 0.4142 | −0.1968 | −0.2095 | 0.3499 | −0.1567 | −0.1906 |
| | 30 | 0.5512 | −0.3874 | 0.4054 | −0.1524 | 0.2682 | 0.0493 | 0.1376 | **0.1451** |
| | 60 | 0.5070 | 0.2607 | 0.7670 | 0.5607 | 1.1131 | 0.6945 | 0.6440 | **0.9880** |
| | 120 | 2.3226 | −0.0740 | 0.5795 | 0.2474 | 2.2021 | 1.4928 | 0.4027 | **0.5526** |
| Quarters | 15 | 0.7499 | 0.4348 | 0.5797 | 0.4745 | 0.4962 | 0.0913 | 0.3779 | 0.1992 |
| | 30 | 1.3705 | 0.2077 | 0.5808 | 0.3852 | 1.0407 | 0.2333 | 0.2572 | 0.1673 |
| | 60 | 0.1659 | 0.0764 | 0.6165 | 0.2746 | 0.9990 | 0.8198 | 0.7180 | 0.4064 |
| | 120 | 1.3905 | −0.0325 | 0.4642 | 0.0558 | 1.3314 | 1.1550 | 0.4419 | **0.4712** |

Note: The table presents the average Information Ratio (return to standard deviation ratio) during the period of the analysis. The 1st scenario assumes that one asset share may range between 0% and 100%, the 2nd scenario limits an asset weight upper-bound to 25%, the 3rd scenario allows for short-selling and relax the previous limitation (asset shares vary between −100% and 100%) and the 4th scenario limits lower and upper bound to −25% and 25%, respectively. *Memory* parameter shows how many historical trading days were used to establish an efficient frontier, whereas *Rebalancing on* parameter presents the frequency of the portfolio reallocation. The bold numbers indicate portfolios, which include the VXX and have higher values of IR.

**Table 7. Annualized average of transaction costs of the Markowitz portfolio**

| Rebalancing on | Memory | 1st scenario (%) | | 2nd scenario (%) | | 3rd scenario (%) | | 4th scenario (%) | |
|---|---|---|---|---|---|---|---|---|---|
| | | no VXX | with VXX | no VXX | with VXX | no VXX | with VXX | no VXX | with VXX |
| Weeks | 15 | 6.79 | 7.14 | 5.73 | 6.29 | 17.85 | 21.84 | 10.19 | 10.68 |
| | 30 | 3.84 | 4.29 | 3.92 | 4.22 | 7.10 | 9.05 | 6.99 | 7.15 |
| | 60 | 2.59 | 2.67 | 2.84 | 2.78 | 4.06 | 5.11 | 4.50 | 4.41 |
| | 120 | 1.10 | 1.20 | 1.96 | 2.85 | 1.69 | 2.28 | 3.05 | 2.86 |
| Months | 15 | 2.94 | 3.01 | 2.3 | 2.53 | 5.13 | 6.54 | 3.77 | 3.79 |
| | 30 | 1.96 | 2.02 | 1.95 | 1.99 | 2.80 | 3.52 | 3.15 | 3.03 |
| | 60 | 1.35 | 1.38 | 1.36 | 1.34 | 1.81 | 2.29 | 2.11 | 2.01 |
| | 120 | 0.71 | 0.75 | 0.92 | 0.84 | 0.88 | 1.35 | 1.48 | 1.38 |
| Quarters | 15 | 1.05 | 0.97 | 0.69 | 0.77 | 1.55 | 2.29 | 1.11 | 1.23 |
| | 30 | 0.68 | 0.74 | 0.70 | 0.70 | 0.87 | 1.51 | 1.21 | 1.25 |
| | 60 | 0.84 | 0.87 | 0.72 | 0.72 | 1.11 | 1.38 | 1.09 | 1.02 |
| | 120 | 0.44 | 0.47 | 0.54 | 0.52 | 0.56 | 0.80 | 0.89 | 0.81 |

Note: The table presents the annualized average of transactional costs during the period of analysis. The 1st scenario assumes that one assets hare may range between 0% and 100%, the 2nd scenario limits an asset weight upper–bound to 25%, the 3rd scenario allows for the short-selling and relax the previous limitation (asset shares vary between –100% and 100%) and the 4th scenario limits the lower and upper bound to –25% and 25%, respectively. Memory parameter shows how many historical trading days were used to establish an efficient frontier, whereas *Rebalancing on* parameter presents the frequency of the portfolio reallocation.

ment results. An analysis of transaction costs may be a useful tool to understand such weak portfolio performance. Table 7 shows the average transaction costs as a percentage. In order to make an intuitive comparison between different frequencies the figures are annualized, i.e. weeks, months and quarters are multiplied by 52, 12 and 4, respectively. The general conclusion is that more frequent rebalancing produces higher expenses and reduces investment profitability. What is more the length of the *memory* parameter results in lower transaction costs which may be explained by more stable weights and a smaller Turnover Ratio. There is also a relatively substantial difference between the first and the third scenario. An allowance for the short selling produces higher transaction costs which was expected as the Turnover Ratio almost doubles.

A long-term analysis of the portfolio selection with the VXX showed that the inclusion of volatility exposure does not systematically improve the performance of the portfolios (except minor examples). The addition of the VIX ETN to the process of portfolio optimization using the Markowitz framework has worsened the Information Ratio when compared to those portfolios without volatility instruments. Even though the VXX has performed poorly during the period of analysis there may exist sub-periods when the incorporation of volatility exposure could bring profitable results.

Tables 8-11 present the year by year analysis of the differences between the Information Ratios for both types of portfolio. Positive values of this difference would indicate the beneficial effect of the VXX inclusion in the given sub-period.

The results for the first scenario follow a similar trend as the analysis for the whole period. The numbers for the second scenario have slightly improved as 39 out of 105 cases have higher IR when the VXX was included. The yearly analysis for the third scenario exhibits a similar percentage of higher Information Ratio for the portfolios which include the VXX as for the second scenario (approximately 40%). However, the pattern is not as clear as previously. The ratio of positive values of the difference for the fourth scenario is the highest among all assumptions and is equal to 48.5%. Similarly, to the second case, higher Information Ratios were obtained by the portfolios with volatility exposure in 2011, 2014, 2015 and 2018.

The general conclusions from the portfolio optimization by means of the Markowitz framework are unambiguous. The inclusion of the VXX into the optimization procedure led to the deterioration of the investment performance, in terms of both annualized average returns and an annualized Information Ratio. Even though the assumption of a range of a single asset from –25% to 25% produced relatively better results, compared to the other scenarios—the Information Ratios were higher only for 50% of cases which cannot provide any strong conclusions about the investment profitability with volatility instruments. Nevertheless, there exist sub-periods when the differences between portfolios with and without the VXX are positive. It may suggest a special treatment of the volatility instrument with precaution and proper investment tools.

**Table 8. Yearly differences of IR between portfolios with and without the VXX for the first scenario**

| Rebalancing on | Memory | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|---|---|---|---|---|---|
| Weeks | **15** | **0.2181** | −2.283 | **0.2107** | −0.2982 | −0.4283 | −0.3314 | −0.3414 | −0.6258 | **0.5557** |
| | **30** | **0.0739** | −1.0344 | −0.4522 | −1.2326 | −0.87 | **0.4709** | −1.2905 | −0.7364 | **0.8049** |
| | **60** | −0.0896 | **0.2242** | −0.211 | −0.5367 | −0.2743 | −0.1034 | −1.925 | −0.0277 | **0.4136** |
| | **120** | NA | −0.5276 | −0.2636 | −0.4097 | −0.0739 | −1.2836 | −0.4242 | −0.0879 | **0.3203** |
| Months | **15** | −2.3963 | −0.5428 | −1.4921 | −1.5669 | **0.2315** | −0.1492 | −0.4108 | −0.0796 | **0.7454** |
| | **30** | **0.0333** | **0.0169** | −0.2702 | −2.0004 | **0.3227** | −0.8034 | −0.4247 | −0.1605 | **0.4146** |
| | **60** | **0.0354** | −0.607 | −0.0804 | −0.9344 | −0.2372 | −0.1876 | −0.3963 | **0.0102** | **0.0236** |
| | **120** | NA | −0.5402 | −0.2263 | −0.1522 | −0.1363 | −1.1116 | −0.5842 | −0.0562 | −0.1396 |
| Quarters | **15** | −0.0721 | **0.0744** | **0.1457** | −1.6843 | −0.7598 | −0.2798 | −0.2194 | −0.0229 | −0.1628 |
| | **30** | **0.0026** | −1.1874 | −0.0476 | −2.2285 | −0.7999 | **1.0696** | −0.7745 | −0.0003 | −0.4082 |
| | **60** | **0.0354** | −0.3993 | −0.0805 | −0.8469 | −0.5463 | −0.0265 | −0.1127 | −0.0032 | −0.0578 |
| | **120** | NA | −0.451 | −0.4582 | −0.1107 | −0.0477 | −0.2574 | −0.3005 | −0.0217 | −0.0158 |

Note: Values above 0, i.e. when portfolio with the VXX has higher Information Ratio than without, are bolded. As the period of analysis starts at the end of September 2010 it is not possible to establish portfolio based on the last 120 trading days in the first sub-period which results in the NA values in the third column. *Memory* parameter shows how many historical trading days were used to establish the efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation.

**Table 9. Yearly differences of IR between portfolios with and without the VXX for the second scenario**

| Rebalancing on | Memory | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|---|---|---|---|---|---|
| Weeks | 15 | -1.7391 | -0.691 | -1.0848 | -0.8193 | 0.2335 | -1.076 | -1.5108 | -1.2514 | 1.7729 |
| | 30 | 0.0163 | 0.1563 | -1.5499 | -1.3867 | -0.3118 | -0.1356 | -1.3463 | -0.7461 | 2.5854 |
| | 60 | -0.0463 | 1.3415 | -0.7305 | -0.936 | 0.5943 | -0.4265 | -0.6111 | -0.2192 | 0.6183 |
| | 120 | NA | 1.3029 | -1.1398 | -0.3235 | 0.3401 | -0.4377 | -0.4376 | -0.0845 | 0.9074 |
| Months | 15 | -5.0145 | 0.7928 | -1.8124 | -1.5234 | -0.0913 | -0.3351 | -0.7465 | -0.1429 | 0.0628 |
| | 30 | 0.0044 | 0.5194 | -2.0935 | -1.9528 | 0.2568 | -0.4807 | -0.186 | -0.1704 | 1.0822 |
| | 60 | 0.0172 | 0.9668 | -0.485 | -1.1956 | 0.0132 | -0.646 | -0.0241 | 0.0205 | 0.0319 |
| | 120 | NA | 1.2856 | -0.9791 | 0.0146 | 0.2865 | -0.4251 | -0.4271 | -0.0647 | 0.6103 |
| Quarters | 15 | -0.0117 | 2.0543 | -0.2023 | -1.961 | 0.305 | -0.3588 | -0.6892 | -0.0331 | -0.3537 |
| | 30 | -0.005 | 0.8084 | -0.1667 | -1.3535 | 0.535 | 1.1203 | -0.4613 | 0.0011 | -0.3385 |
| | 60 | 0.0172 | -0.4356 | -0.8377 | -0.7354 | 0.0344 | -0.4909 | 0.0504 | 0.0118 | -0.4532 |
| | 120 | NA | 0.2124 | -1.3429 | -0.3951 | 0.2018 | -0.3904 | -0.9123 | -0.0596 | 0.4305 |

Note: Values above 0, i.e. when portfolio with the VXX has higher Information Ratio than without, are bolded. As the period of analysis starts at the end of September 2010 it is not possible to establish portfolio based on the last 120 trading days in the first sub-period which results in the NA values in the third column. *Memory* parameter shows how many historical trading days were used to established efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation.

**Table 10. Yearly differences of IR between portfolios with and without the VXX for the third scenario**

| Rebalancing on | Memory | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|---|---|---|---|---|---|
| Weeks | 15 | **2.535** | –1.063 | –0.161 | –0.4715 | **0.6006** | **0.364** | **0.8519** | –0.0206 | **0.6232** |
| | 30 | **0.4358** | –0.6254 | **0.0791** | –1.1454 | –0.0806 | –0.3939 | **0.5156** | –0.4851 | **0.9003** |
| | 60 | **0.0235** | **0.4157** | –0.8258 | –0.6708 | **0.254** | **0.1624** | –0.294 | **0.0725** | **0.1166** |
| | 120 | NA | –0.07 | –0.0377 | –0.2684 | **0.0719** | –1.7112 | –0.4027 | –0.1148 | **0.2651** |
| Months | 15 | –0.5229 | **0.0056** | –0.4907 | –1.294 | **0.3832** | –0.256 | **0.7657** | –0.8157 | **0.5159** |
| | 30 | –0.3208 | **0.0188** | **0.3722** | –0.0271 | **0.1273** | –0.3378 | –0.166 | –0.372 | –0.4794 |
| | 60 | **0.0732** | –0.2421 | –0.1797 | –0.2941 | **0.1282** | –0.175 | –0.4128 | –0.0745 | –0.0071 |
| | 120 | NA | –0.6669 | –0.1216 | –0.107 | **0.0096** | –0.736 | –1.1011 | –0.066 | **0.0017** |
| Quarters | 15 | **0.3712** | –1.8074 | –0.21 | –0.2532 | –0.1501 | –0.6771 | **0.2776** | –0.3157 | **0.1632** |
| | 30 | –0.245 | –0.3767 | **0.4966** | –1.5145 | –0.6109 | **0.1863** | –0.7637 | **0.0099** | –1.2379 |
| | 60 | **0.0732** | –1.3095 | –0.1516 | –0.325 | –0.2362 | **1.264** | –0.7187 | –0.1294 | –0.1505 |
| | 120 | NA | –0.6851 | –0.2911 | –0.2186 | **0.1118** | **0.8869** | –0.9294 | **0.0418** | **0.181** |

Note: Values above 0, i.e. when portfolio with the VXX has higher Information Ratio than without, are bolded. As the period of analysis starts at the end of September 2010 it is not possible to establish portfolio based on the last 120 trading days in the first sub-period which results in the NA values in the third column. *Memory* parameter shows how many historical trading days were used to established efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation.

**Table 11. Yearly differences of IR between portfolios with and without the VXX for the fourth scenario**

| Rebalancing on | Memory | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|---|---|---|---|---|---|
| Weeks | 15 | -0.6691 | -0.3982 | -0.4605 | -0.7607 | **0.8592** | **0.3403** | -0.2319 | -0.5581 | **1.0518** |
|  | 30 | -0.3083 | **0.4264** | -0.2198 | -1.2213 | **0.0841** | **0.2312** | -0.787 | -0.4083 | **1.1461** |
|  | 60 | **2.9884** | **1.217** | -0.3108 | -0.0451 | **0.5335** | **0.111** | -0.5215 | **0.1547** | **0.8696** |
|  | 120 | NA | **1.7183** | -1.2747 | **0.0073** | **0.7227** | **0.3414** | **0.0487** | -0.0691 | -0.0657 |
| Months | 15 | -2.8274 | **1.3453** | -1.0986 | -0.4819 | **0.5177** | **0.2017** | **0.0553** | -0.7074 | **0.8748** |
|  | 30 | -1.0239 | **0.8989** | -0.5414 | -0.9699 | **0.4918** | -0.2038 | -0.0565 | -0.0378 | **0.1023** |
|  | 60 | **2.6155** | **1.6443** | **0.2111** | -0.1372 | **0.4196** | -0.1367 | -0.2552 | **0.1557** | **0.6071** |
|  | 120 | NA | **1.9655** | -1.2465 | **0.1787** | **0.3097** | **0.1527** | -0.2265 | -0.0311 | **0.1939** |
| Quarters | 15 | -0.2212 | **1.6826** | -0.6129 | -1.3035 | -0.0588 | **0.2302** | -0.6292 | -0.1025 | -0.0819 |
|  | 30 | **1.581** | **1.0765** | **0.2159** | -0.8814 | -0.5181 | **0.6556** | **0.04** | -0.0663 | -1.2786 |
|  | 60 | **2.6155** | **0.3424** | -0.9576 | -0.4205 | -0.6966 | -0.3328 | **0.1433** | -0.1527 | -0.8883 |
|  | 120 | NA | **0.4051** | -0.8779 | -0.2408 | **0.1758** | **0.1503** | -0.1253 | -0.1732 | **0.2074** |

Note: Values above 0, i.e. when portfolio with the VXX has higher Information Ratio than without, are bolded. As the period of analysis starts at the end of September 2010 it is not possible to establish portfolio based on the last 120 trading days in the first sub-period which results in the NA values in the third column. *Memory* parameter shows how many historical trading days were used to established efficient frontier, whereas *Rebalancing on* parameter presents the frequency of portfolio reallocation.

## 4.2. Naïve diversification

Tables 12 and 13 present the results of annual return compounded and Information Ratio for a naïve diversification approach. The clear pattern may

**Table 12. Annual return compounded (ARC) for portfolios with fixed weights (%)**

| Scenario | Rebalancing on | No VXX | VXX 1% | VXX 3% | VXX 5% |
|---|---|---|---|---|---|
| First | weeks | 3.33 | 2.67 | 1.37 | 0.06 |
| | months | 3.34 | 2.68 | 1.38 | 0.08 |
| | quarters | 3.38 | 2.81 | 1.68 | 0.54 |
| Second | weeks | 3.46 | 2.82 | 1.55 | 0.22 |
| | months | 3.48 | 2.83 | 1.55 | 0.23 |
| | quarters | 3.53 | 2.98 | 1.89 | 0.73 |
| Third | weeks | 3.80 | 3.14 | 1.85 | 0.51 |
| | months | 3.83 | 3.16 | 1.85 | 0.50 |
| | quarters | 3.86 | 3.31 | 2.21 | 1.06 |
| Fourth | weeks | 6.30 | 5.62 | 4.31 | 2.95 |
| | months | 6.33 | 5.65 | 4.31 | 2.94 |
| | quarters | 6.36 | 5.80 | 4.68 | 3.50 |

Note: The table presents the annualized average of returns for portfolios with fixed weights. *Rebalancing on* parameter presents how often the weights of portfolio return to their initially fixed values. The detailed description of scenarios and their assumptions are included in the Methodology section.

**Table 13. Annualized Information Ratio for portfolios with fixed weights**

| Scenario | Rebalancing on | No VXX | VXX 1% | VXX 3% | VXX 5% |
|---|---|---|---|---|---|
| First | weeks | 0.7078 | 0.6092 | 0.3351 | 0.0146 |
| | months | 0.7157 | 0.6180 | 0.3407 | 0.0180 |
| | quarters | 0.7275 | 0.6509 | 0.4167 | 0.1257 |
| Second | weeks | 0.3930 | 0.3323 | 0.1941 | 0.0288 |
| | months | 0.3970 | 0.3363 | 0.1970 | 0.0298 |
| | quarters | 0.4044 | 0.3558 | 0.2407 | 0.0967 |
| Third | weeks | 0.2448 | 0.2073 | 0.1274 | 0.0364 |
| | months | 0.2466 | 0.2089 | 0.1282 | 0.0361 |
| | quarters | 0.2487 | 0.2188 | 0.1538 | 0.0766 |
| Fourth | weeks | 0.4677 | 0.4344 | 0.3592 | 0.2641 |
| | months | 0.4707 | 0.4376 | 0.3624 | 0.2661 |
| | quarters | 0.4732 | 0.4495 | 0.3938 | 0.3177 |

Note: The table presents the annualized average of Information Ratio for portfolios with fixed weights. *Rebalancing on* parameter presents how often the weights of portfolio return to their initially fixed values. The detailed description of scenarios and their assumptions are included in the Methodology section.

**Table 14. Annualized average of the transactional costs [%] for portfolios with fixed weights**

| Scenario | Rebalancing on | No VXX | VXX 1% | VXX 3% | VXX 5% |
|---|---|---|---|---|---|
| First | weeks | 0.16 | 0.16 | 0.18 | 0.19 |
| | months | 0.11 | 0.11 | 0.12 | 0.13 |
| | quarters | 0.08 | 0.08 | 0.09 | 0.10 |
| Second | weeks | 0.19 | 0.19 | 0.21 | 0.23 |
| | months | 0.12 | 0.12 | 0.13 | 0.14 |
| | quarters | 0.09 | 0.09 | 0.10 | 0.10 |
| Third | weeks | 0.15 | 0.16 | 0.18 | 0.20 |
| | months | 0.10 | 0.11 | 0.12 | 0.13 |
| | quarters | 0.08 | 0.08 | 0.09 | 0.10 |
| Fourth | weeks | 0.17 | 0.18 | 0.20 | 0.23 |
| | months | 0.12 | 0.12 | 0.13 | 0.14 |
| | quarters | 0.09 | 0.09 | 0.10 | 0.10 |

Note: The table presents the annualized average of Information Ratio for portfolios with fixed weights. *Rebalancing on* parameter presents how often the weights of portfolio return to their initially fixed values. The detailed description of scenarios and their assumptions are included in the Methodology section.

be observed for all scenarios: higher volatility exposure results in a decrease in profitability.

The tables confirm that the negative average return of volatility ETN decreases the efficiency of the portfolios. The first and the second scenario have similar values of the ARC while IR is much better for the first scenario. Taking into account the third and the fourth scenario the results are much better when the foreign assets have a larger share. Both annualized returns and Information Ratio have higher values for the fourth scenario. Nevertheless, the lower the share of the VXX, the better the overall performance.

The more often frequency of rebalancing results in slightly poorer performance of the portfolios which finds the explanation in Table 14 where annualized transaction costs are displayed. Even though the average of Turnover Ratio is smaller for weekly rebalancing the annualized figures show that the more frequent rebalancing contributes to higher transaction costs every year. What is more a rise of the VXX share in portfolio results in higher transaction costs too. It may be explained by the fact that the VXX is one of the most volatile assets within the portfolio which consequently increases the Turnover Ratio and costs.

Tables 15-18 present the Information Ratio on a yearly basis for each scenario. The detailed analysis of each sub-period may provide some extra information about the portfolio performance and its more effective management.

**Table 15. Yearly values of Information Ratio for portfolio with fixed weights. First scenario**

| Rebalancing on | VXX Share | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|---|---|---|---|---|---|
| Weeks | No VXX | **2.531** | -0.2102 | **2.5277** | **0.4705** | 0.7936 | **-0.737** | **0.7207** | **3.0946** | -0.1794 |
| | VXX 1% | 2.1058 | -0.185 | 2.3497 | 0.23 | **0.8173** | -0.8287 | 0.5497 | 2.7529 | -0.0568 |
| | VXX 3% | 1.0346 | -0.1231 | 1.7607 | -0.2908 | 0.7796 | -0.9669 | 0.1361 | 1.8578 | 0.1869 |
| | VXX 5% | -0.2604 | **-0.0396** | 0.9799 | -0.7886 | 0.6355 | -0.9877 | -0.2925 | 0.8931 | **0.3329** |
| Months | No VXX | **2.5341** | -0.2229 | **2.5448** | **0.4792** | 0.7919 | **-0.7514** | **0.7183** | **3.0844** | -0.1652 |
| | VXX 1% | 2.1338 | -0.1938 | 2.3792 | 0.2436 | **0.792** | -0.8363 | 0.5417 | 2.7399 | -0.0821 |
| | VXX 3% | 1.1462 | -0.1181 | 1.8283 | -0.2669 | 0.702 | -0.955 | 0.1107 | 1.8397 | 0.0906 |
| | VXX 5% | -0.0622 | **-0.016** | 1.0817 | -0.762 | 0.5186 | -0.9504 | -0.3257 | 0.8801 | **0.1962** |
| Quarters | No VXX | **2.5435** | -0.2019 | **2.5335** | **0.4519** | **0.8127** | **-0.7444** | **0.7521** | **3.0048** | -0.1667 |
| | VXX 1% | 2.2073 | -0.0928 | 2.3918 | 0.2299 | 0.8078 | -0.8685 | 0.5741 | 2.6993 | -0.0482 |
| | VXX 3% | 1.4028 | 0.1842 | 1.901 | -0.2449 | 0.7073 | -1.075 | 0.1461 | 1.9204 | 0.1744 |
| | VXX 5% | 0.3988 | **0.4816** | 1.2088 | -0.7159 | 0.5142 | -1.1481 | -0.2948 | 1.0629 | **0.2926** |

Note: In each sub-period and each frequency of rebalancing, 4 different portfolios are considered: with 1%, 3%, and 5% of the VXX share and without volatility exposure at all (*VXX Share* parameter). The portfolio with the highest Information Ratio in each year is bolded. *Rebalancing on* parameter presents how often the weights of portfolio return to their initially fixed values.

**Table 16. Yearly values of Information Ratio for portfolio with fixed weights. Second scenario**

| Rebalancing on | VXX Share | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|---|---|---|---|---|---|
| Weeks | No VXX | **2.6051** | **−0.5712** | **2.0335** | **0.2562** | **0.3434** | **−0.7443** | **0.5193** | **2.4322** | −0.3512 |
| | VXX 1% | 2.3557 | −0.5756 | 1.918 | 0.1229 | 0.3376 | −0.7929 | 0.4259 | 2.2497 | −0.2904 |
| | VXX 3% | 1.7718 | −0.5818 | 1.6249 | −0.157 | 0.3239 | −0.8781 | 0.2175 | 1.8353 | −0.1562 |
| | VXX 5% | 1.1154 | −0.5789 | 1.2503 | −0.4634 | 0.2759 | −0.964 | −0.0085 | 1.3558 | **−0.0196** |
| Months | No VXX | **2.6046** | **−0.5861** | **2.0457** | **0.2656** | **0.3387** | **−0.7527** | **0.5169** | **2.4303** | −0.3408 |
| | VXX 1% | 2.3692 | −0.5919 | 1.9383 | 0.1342 | 0.32 | −0.7989 | 0.4201 | 2.2456 | −0.3005 |
| | VXX 3% | 1.8249 | −0.5992 | 1.6652 | −0.1412 | 0.2787 | −0.8789 | 0.2029 | 1.8276 | −0.2057 |
| | VXX 5% | 1.2204 | −0.5932 | 1.3142 | −0.4434 | 0.204 | −0.9582 | −0.0326 | 1.3454 | **−0.1026** |
| Quarters | No VXX | **2.6131** | −0.5772 | **2.0367** | **0.2481** | **0.351** | **−0.7487** | **0.5404** | **2.3793** | −0.3416 |
| | VXX 1% | 2.416 | −0.5447 | 1.9432 | 0.1239 | 0.3293 | −0.8141 | 0.4425 | 2.2138 | −0.2823 |
| | VXX 3% | 1.965 | −0.464 | 1.7014 | −0.1331 | 0.2801 | −0.9354 | 0.2251 | 1.8458 | −0.1492 |
| | VXX 5% | 1.4714 | **−0.3563** | 1.3837 | −0.4165 | 0.1995 | −1.0574 | −0.0098 | 1.4236 | **−0.0176** |

Note: In each sub-period and each frequency of rebalancing, 4 different portfolios are considered: with 1%, 3%, and 5% of the VXX share and without volatility exposure at all (*VXX Share* parameter). The portfolio with the highest Information Ratio in each year is bolded. *Rebalancing on* parameter presents how often the weights of portfolio return to their initially fixed values.

**Table 17. Yearly values of Information Ratio for portfolio with fixed weights. Third scenario**

| Rebalancing on | VXX Share | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|---|---|---|---|---|---|
| Weeks | No VXX | **2.7638** | **−0.7261** | **1.6188** | **0.2693** | **0.1155** | **−0.6604** | **0.4187** | **2.2344** | −0.4565 |
| | VXX 1% | 2.5962 | −0.7326 | 1.5409 | 0.1853 | 0.1046 | −0.688 | 0.3642 | 2.1213 | −0.4258 |
| | VXX 3% | 2.2438 | −0.7429 | 1.3671 | 0.0131 | 0.084 | −0.7461 | 0.2506 | 1.8798 | −0.3543 |
| | VXX 5% | 1.8555 | −0.7547 | 1.1668 | −0.1717 | 0.0535 | −0.8024 | 0.1255 | 1.6156 | **−0.2788** |
| Months | No VXX | **2.7587** | **−0.7277** | **1.6195** | **0.2798** | **0.1133** | **−0.6587** | **0.4198** | **2.2418** | −0.4538 |
| | VXX 1% | 2.6001 | −0.7368 | 1.547 | 0.196 | 0.0941 | −0.6854 | 0.3631 | 2.1264 | −0.4357 |
| | VXX 3% | 2.2693 | −0.7519 | 1.3855 | 0.0248 | 0.0567 | −0.742 | 0.244 | 1.8803 | −0.3879 |
| | VXX 5% | 1.9071 | −0.7674 | 1.1988 | −0.1585 | 0.0093 | −0.797 | 0.1127 | 1.6117 | **−0.3338** |
| Quarters | No VXX | **2.7608** | −0.73 | **1.6199** | **0.2768** | **0.1169** | **−0.655** | **0.4305** | **2.2207** | −0.4519 |
| | VXX 1% | 2.6279 | −0.7188 | 1.5558 | 0.1964 | 0.0953 | −0.6922 | 0.3728 | 2.1163 | −0.423 |
| | VXX 3% | 2.3536 | −0.6897 | 1.4119 | 0.0336 | 0.0529 | −0.7701 | 0.2526 | 1.8959 | −0.3519 |
| | VXX 5% | 2.0538 | **−0.6566** | 1.2432 | −0.1405 | 0.0006 | −0.8469 | 0.1211 | 1.6571 | **−0.2749** |

Note: In each sub-period and each frequency of rebalancing, 4 different portfolios are considered: with 1%, 3%, and 5% of the VXX share and without volatility exposure at all (*VXX Share* parameter). The portfolio with the highest Information Ratio in each year is bolded. *Rebalancing on* parameter presents how often the weights of portfolio return to their initially fixed values.

**Table 18. Yearly values of Information Ratio for portfolio with fixed weights. Fourth scenario**

| Rebalancing on | VXX Share | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|---|---|---|---|---|---|
| Weeks | No VXX | **3.072** | **−0.5847** | **1.581** | **1.0471** | **0.2833** | **−0.3332** | **0.4912** | **2.7951** | −0.2106 |
|  | VXX 1% | 2.9198 | −0.5896 | 1.5093 | 0.9664 | 0.2792 | −0.3584 | 0.4347 | 2.6833 | −0.1675 |
|  | VXX 3% | 2.5923 | −0.5972 | 1.3428 | 0.7895 | 0.2724 | −0.4164 | 0.3106 | 2.4214 | −0.0623 |
|  | VXX 5% | 2.2106 | −0.6083 | 1.139 | 0.5751 | 0.2512 | −0.4797 | 0.1618 | 2.0967 | **0.0534** |
| Months | No VXX | **3.0612** | **−0.5857** | **1.5827** | **1.0664** | **0.2815** | **−0.3325** | **0.4933** | **2.8073** | −0.2061 |
|  | VXX 1% | 2.9161 | −0.5934 | 1.5157 | 0.9867 | 0.2671 | −0.3545 | 0.4346 | 2.6934 | −0.1817 |
|  | VXX 3% | 2.6077 | −0.6068 | 1.3606 | 0.813 | 0.238 | −0.4055 | 0.3044 | 2.4262 | −0.1169 |
|  | VXX 5% | 2.2526 | −0.6238 | 1.1705 | 0.6028 | 0.1925 | −0.4617 | 0.1469 | 2.0951 | **−0.0442** |
| Quarters | No VXX | **3.0637** | −0.5881 | **1.5838** | **1.0586** | **0.2854** | **−0.3237** | **0.508** | **2.7738** | −0.201 |
|  | VXX 1% | 2.9425 | −0.5717 | 1.5265 | 0.9812 | 0.2684 | −0.359 | 0.4477 | 2.6697 | −0.1633 |
|  | VXX 3% | 2.689 | −0.5316 | 1.3925 | 0.8152 | 0.2342 | −0.4389 | 0.3158 | 2.4307 | −0.0672 |
|  | VXX 5% | 2.4002 | **−0.4872** | 1.2256 | 0.6173 | 0.183 | −0.5264 | 0.1586 | 2.1405 | **0.0391** |

Note: In each sub-period and each frequency of rebalancing, 4 different portfolios are considered: with 1%, 3%, and 5% of the VXX share and without volatility exposure at all (*VXX Share* parameter). The portfolio with the highest Information Ratio in each year is bolded. *Rebalancing on* parameter presents how often the weights of portfolio return to their initially fixed values.

The results for the first scenario show that the incorporation of the VXX reduces risk-adjusted return for all years except 2011, 2014 and 2018. The results for the second, the third and the fourth scenarios adhere to the following pattern: the volatility exposure is profitable in 2018 as the share of the VXX equal to 5% brings the highest risk-adjusted return (in most of the cases it reduces losses instead of providing extra positive return).

For all remaining cases volatility instruments do not improve the yearly performance by increasing the Information Ratio.


## Conclusions

The introduction of the VIX index and its derivatives allowed for new investment possibilities. A negative correlation with most of the markets encouraged investors to seek and exploit new strategies which include exposure to volatility. The non-standard characteristics of the VIX index were considered to be beneficial for long-term investors who are looking for better portfolio diversification. However, the detailed research showed that it is not always valid and significantly depends on the markets' conditions. The global financial crisis in 2008 and the euro debt crisis in 2011 are good examples of when investors may benefit from this new asset class as the volatility index jumped significantly and provided impressive profits. Nonetheless, the long-term tendency for volatility instruments is characterized by a negative expected return caused by very high average roll yields which decrease the overall profits.

The research aimed to construct the portfolios according to two different frameworks (the mean-variance of Markowitz and the naïve diversification with fixed weights) and compare the risk-adjusted return between the portfolios with and without the volatility exposure. As many studies have evaluated the diversification issues from the American and European perspective already this analysis focused on the Polish investor viewpoint. To benefit from volatility as an asset class that investor had to look for the foreign opportunities as the Warsaw Stock Exchange does not provide the necessary instruments. The VXX was chosen as the proxy for volatility due to its long data availability and a common recognition as the most popular volatility ETN. The final portfolio contained a set of different asset classes from various locations.

The general conclusions are mostly unambiguous. The Polish investor faces similar problems as the American, European and global ones as the portfolios which included the volatility exposure performed generally worse than those without the VXX in the long-term. The only example when the overall performance was relatively better is the portfolio optimized by means of the Markowitz framework when short selling is allowed and the weight of a single asset varies from –25 to 25. However, these results were only better when

compared to other scenarios as the Information Ratio was higher in only 50 of cases which cannot produce any durable conclusions.

Referring to the main research question it can be stated that a hypothetical investor who has a diversified portfolio cannot improve his efficiency, measured by risk-adjusted returns, by an inclusion of volatility exposure. The results for the naïve diversification approach (the second research question) confirmed the outcomes of Markowitz procedure—the VXX inclusion worsened the investment profitability and performance (i.e. risk-adjusted ratios). The year by year analysis, however, showed that some improvements in the portfolio outcomes may be achieved. The higher Information Ratios for the portfolios with volatility exposure were observed in 2011, 2014 and 2018 but it much depended on selected criteria (i.e. the frequency of rebalancing and the length of the memory parameter).

The detailed sensitivity analysis performed in the fourth section did not change the initial conclusions (the third research question). Various values of the rebalancing period, the memory parameter, or different weights of assets produced similar results.

A chosen period is a crucial factor for the above results. The analysis covers the data from the end of 2010 until mid 2018 when no severe downturn took place. The euro debt crisis in 2011, a serious decrease in oil price in 2014 or the Brexit referendum in 2016 are the examples of when the markets were hit by uncertainty. However, the scale of these events was much less significant when compared to the banking crisis of 2008 or the dot-com bubble and the scepticism of investors about the market's perspective was only temporary. Most of the equity assets exhibited steady growth, stimulated by economic expansion and monetary policy. The results confirm that the volatility exposure in the portfolio may be profitable in sub-periods (e.g. 2011, 2018) but profits are generally deteriorated in the long-term unless a significant financial crash occurs. Prediction of a financial crisis, however, seems to be an extremely difficult task that goes beyond the aim of a passive long-term investor.

Another important factor that affected the obtained results is related to VIX futures market specifics. As Alexander and Korovilas (2012) have shown keeping permanent volatility exposure to VIX futures incurs high rolling costs due to the upward slope of the VIX futures term structure by most of the time. The VXX invests in short-term contracts and as a result, is particularly exposed to the high costs of carry. This aspect is linked with a durable difference between an implied and realized volatility (volatility risk premium—VRP). Investors tend to overshoot implied volatility when compared to ex-post realized value. As Asensio (2013) presented, the VRP of VIX futures is permanent (and particularly intensive in the post-crisis period) and especially visible for the short-term contracts. The author concluded that higher interest in volatility instruments, by both professional and individual investors, results in greater mispricing as more participants are exposed to the forecast bias. It is also mentioned

that VIX futures are treated as insurance for unfavourable market movements which may push the VRP even more. Summarizing the volatility risk premium may contribute to the poorer investment results for the long-term investors who are interested in a long volatility exposure.

Further research can be dedicated to improving the portfolio optimization procedure. This analysis has not considered higher order moments of returns or the downside risk. Besides, different maximising functions can be chosen to consider investor preferences. What is more, following the idea proposed by Ślepaczuk and Zakrzewski (2008) of an introduction of the volatility index based on WIG20 options, the potential diversification benefits for Polish equity market may be much higher with the VIW20 index instead of VIX index.

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## Aims and Scope

The **Economics and Business Review** is a quarterly journal focusing on theoretical, empirical and applied research in the fields of Economics and Corporate and Public Finance. The Journal welcomes the submission of high quality articles dealing with micro, mezzo and macro issues well founded in modern theories and relevant to an international audience. The EBR's goal is to provide a platform for academicians all over the world to share, discuss and integrate state-of-the-art Economics and Finance thinking with special focus on new market economies.

## The manuscript

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